

show more than one component, and so are more like the normal experimental curves. As yet we can only claim that the simulations do produce curves in qualitative agreement with typical experimental ones. It is hoped to publish more detailed physiological results elsewhere.

References

- RICHARDS, T. G., TINDALL, V. R., and YOUNG, A. (1959). "A modification of the Bromsulphthalein liver function test to predict the dye content of the liver and bile," *Clin. Sci.*, Vol. 18, p. 199.
- BARBER-RILEY, G., GOETZEE, A. E., RICHARDS, T. G., and THOMSON, J. Y. (1961). "The transfer of Bromsulphthalein from the plasma to the bile in man," *Clin. Sci.*, Vol. 20, p. 149.
- GANTMACHER, F. R. (1959). *The Theory of Matrices*, Vol. 2 (New York: Chelsea Publishing Co.).
- SHERLOCK, S., BEARN, A. G., BILLING, B. H., and PATERSON, J. C. S. (1950). "Splanchnic blood in man by the bromsulphthalein method: The relation of peripheral plasma bromsulphthalein level to the calculated flow," *J. Lab. Clin. Med.*, Vol. 35, p. 923.

Book Reviews

Discrete Variable Methods in Ordinary Differential Equations, by PETER HENRICI, 1962; 407 pages. (London: John Wiley and Sons Ltd., 87s. 0d.)

A discrete variable method is one which provides approximations to the values of exact solutions at a set of lattice points defined by discrete values of the independent variable. Accordingly, this book is concerned mainly with Runge-Kutta and finite-difference methods for initial-value problems, and with finite-difference methods for boundary-value problems. The central theme is analysis of the errors in the numerical solutions due to departures from the correct Taylor expansions, and to round-off.

For the solution of initial-value problems, methods are classified as *one-step* when the approximation at one lattice point depends on information at only the immediately preceding lattice point (e.g. the Euler and Runge-Kutta methods), and as *multistep* when information at more than one of the preceding lattice points is required (e.g. the Adams-Bashforth and Milne-Simpson methods). Chapters 1 to 4 concern one-step methods for single equations of the first and higher orders, and for systems thereof. Multistep methods for first order equations are discussed in Chapter 5, while Chapter 6 concerns their application to equations of the second order from which the first derivative is absent. Finally, Chapter 7 contains material on a class of non-linear boundary-value problems of the second order.

For each class of methods, results are proved concerning existence and convergence (including a version of Kantorovich's theorem on Newton's method for systems of non-linear algebraic equations), and theorems based on a statistical theory of round-off are supported by the results of novel experiments. One interesting conclusion is that "the truncation error is generally not increased, and the round-off error is frequently substantially decreased when an equation of order above the first is reduced to a first order system" (page 109). The recurrent device of constructing auxiliary differential equations which are satisfied by the various errors is also noteworthy.

The book is intended to be both "an up-to-date presentation of significant theoretical results" and "a teaching tool on the senior level." Readers will surely agree that it is wholly successful on each account, indeed that it constitutes a landmark in the literature. The manner of development of the argument is itself an encouragement to the reader: the

We wish to acknowledge the assistance of Miss G. Almond who wrote some of the preliminary trial programs for the computer, and also supervised the running of the simulation program. We also wish to thank Dr. T. G. Richards for his assistance with points of physiological detail.

same pattern of attack is followed in each chapter, and the basic ideas are illustrated by simple examples throughout. The style is clear, the internal cross references are good, the bibliography is extensive, and a hundred and seventy problems for solution are provided. On the other hand, the complete absence of reference to the use of trees and graphs in classifying integration methods is disappointing.

In a wider context, Professor Henrici shows how, at the university, numerical analysis could illustrate the coherent application of what the student may otherwise feel are the unrelated fragments of undergraduate pure mathematics. In the book, the elements of real and complex variable theory and linear algebra are invoked frequently, and the statistical theory of rounding errors offers a stimulating use for arguments usually restricted to coin-tossing and dice-shaking.

D. W. MARTIN.

Mathematics and Industry, by JOHN CRANK, 1962; 91 pages. (London: Oxford University Press, 12s. 6d.)

The above book is the second of the *Oxford Mathematical Handbooks*, written by one of the editors of the series, the Head of the Mathematics Department at Brunel College of Technology. It is intended for Dip. Tech. students but will also be of interest to other students and to sixth-form teachers. The title is a little misleading as it suggests a more comprehensive work which would cover most aspects in a very large field.

The earlier chapters of the book are general and open with a discussion of the uses of mathematics, emphasizing the importance of extracting the mathematical model from the industrial problem before using the normal methods of solution. The second chapter discusses mathematical models in more detail and gives illustrations showing how different levels of approximation affect the solution used. The "flutter" of aeroplanes is used to show how a compromise had to be made between close approximations and convenience of solution, but also illustrates the increased power of modern methods using computers. Dr. Crank appeals for problems designed to "test the student's ability to apply fundamental principles . . . in preference to those which test only his manipulative powers in pure mathematics," and concludes the chapter with eleven problems for the student. The next

(Continued on p. 237.)

with those in (5) and (7) up to $N + 2$, we obtain the coefficients listed in Table 1. For $L = 4$ we have to consider two correction terms $\alpha_1 H_2$ and $\alpha_2 H_{22}$ and equate coefficients up to order $N + 4$. This method can, with some loss of simplicity, be extended to $L = 5$ and 6.

The general formulas I_{LMN} can similarly be derived symbolically if the differential operators D^m are expanded in terms of $\mu\delta^n$; a list of these expansions is given in Chapter 9 of Kopal's book (1961), where the operator 2θ corresponds to the more widely used D , and in the introduction to Miller's report (1960).

References

- KOPAL, Z. (1961). *Numerical Analysis*, Chapman and Hall, London, 2nd edition.
 LANCZOS, C. (1957). *Applied Analysis*, Pitman, London.
 MILLER, J. C. P. (1960). "Quadrature in Terms of Equally-Spaced Function Values," MRC Technical Summary Report 167, University of Wisconsin, Madison.
 SALZER, H. E. (1955). "Osculatory Quadrature Formulas," *J. Maths and Physics*, Vol. 34, pp. 103-112.
 SALZER, H. E., SHOULTZ, D. C., and THOMPSON, E. P. (1960). "Tables of Osculatory Integration Coefficients," Convair (Astronautics) Division of General Dynamics Corporation.

Book Reviews (continued from p. 227)

chapter shows the generality of solutions using dimensionless variables, not only in reducing the work in a particular problem but also for showing the connections between problems from different fields, such as mechanics and electricity, and so making possible the development of analogue computers.

The later chapters in the book are more specific and inevitably reflect the author's interests. A more comprehensive set of illustrations would have needed a team of writers covering other fields, but would have given a more balanced picture of the relations between mathematics and industry. Chapter 4 discusses "spinning in the textile industry," and is followed by one on "Nuclear Reactors," which shows how the partial differential equations are replaced by a large number of simultaneous linear algebraic equations which are adapted to computer solution. Chapter 6 discusses linear programming, an important part of operational research, using the "stepping stone" method and that of "fictitious costs." Chapter 7 shows how much work in aircraft design is solving equations with different boundary conditions corresponding to different profiles, and then discusses how computers have made possible the solutions of the equations of meteorology. The last chapter gives the mathematical analysis of burning wood and relates the analysis to the physics of the process.

Most chapters conclude with one or more references to other books or articles, but a more comprehensive bibliography which could have included other topics from, e.g., the electrical industry, would have been useful. As part of the aim of a series like this is to encourage students to use books, it is unfortunate that no index has been included. Despite this minor criticism the book will be found useful by that increasing body of teachers aware of the importance of relating their mathematics teaching to the problems of modern industry. It should certainly be ordered for all school and college libraries, and many teachers will want to buy a personal copy.

P. J. WALLIS.

Symbols, Signals and Noise, by J. R. PIERCE, 1962; 305 pages. (London: Hutchinson and Co., 21s. 0d.)

It is a pleasure to be able to commend a book as highly as this one can be commended. It is an excellent presentation of, and commentary on, the subject of information theory in its various aspects.

After a preamble on the nature of physical and mathematical theories, the book proceeds through the history of electrical communication, the problems of coding, the idea of measuring information and the problem of combating disturbances. It then discusses the application of these ideas in physics, computing, psychology and art, and ends with a brief mention of some recent investigations.

The author is in a senior position in the Bell Telephone Laboratories and has obviously been closely involved in a great deal of this subject. His historical account, which begins with the origin of the Morse code and with the problem of signalling over telegraph cables, is written with obvious human feeling. This is not a mathematical book, and the mathematical reader may find some of the treatment tedious. It is, however, very definitely a pro-mathematical book, in that it stresses throughout the dominating part played by mathematics in the development of the subject, and tries with a good deal of success to give the reader a feel for the things that motivate mathematicians. The author takes great care to indicate just how far he has attempted to present the subject with precision, and at what point he reverts to a description in general terms. He goes out of his way to help the non-mathematical reader to follow as much as possible of the book, even to the extent of giving a miniature textbook of mathematics in an appendix. (This is in fact probably too brief for its purpose, and is interesting rather as a glimpse of mathematics from a new angle for those who can understand it already.) A more useful appendix is a short but excellent glossary.

Many chapters begin with seemingly irrelevant and wasteful digressions, but these always turn out to be written with a worth-while objective. One even finds that the apparently casual examples in the early chapters are designed to prepare the ground for later chapters in the book. The end of each chapter is a clear summary of its main points. In the later chapters, which roam over such fields as psychology and cybernetics, there is much that will interest and intrigue the reader who is concerned with computers; here, however, the connection with information theory often becomes tenuous. It is clear that the simple matter of measuring amounts of information, which is the central concern of information theory, is only a beginning to a study of these other fields.

(Continued on p. 245)