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Book reviews—Analogue computers

Analogue Computing at Ultra-high Speed, by D. M. MacKay and M. E. Fisher, 1962; 395 pages. (London: *Chapman and Hall Ltd.*, 65s.)

Until recently the development of analogue computation was somewhat in the doldrums. Exigencies of the last world war were the cause of its establishment as a reliable and quick means of solving ordinary differential equations; thereafter the development consisted in the perfecting of the equipment, culminating in the appearance on the world market of general-purpose analogue computers that were accurate and versatile. However, manufacturers and independent researchers began to see hitherto latent possibilities of extending the usefulness of the analogue computer by the incorporation into it of digital devices, such as logic-elements for control and memory-elements for the storage of information, together with the facility of high-speed repetitive operation. Indeed, such hybrid computers have now appeared on the market.

Meanwhile, in fact since 1946, the authors of this book, physicists, had been engaged on research on this very topic. The book is an exposition of their work; as they point out, the bulk of the contents was first described in their Ph.D. theses, and as is to be expected it is written at an advanced level and excludes material on conventional analogue computers.

The book is divided into three parts with a final chapter headed "conclusions." The first and shortest part surveys the problem of computing with reference to information theory. In this an abstract analysis, of which part was developed by one of the authors, is used with some simplifying assumptions to obtain criteria for the efficiency of a computer as an information channel. This is followed by a discussion of the instrumental requirements of a high-speed analogue computer in relation to the types of computational problem to be tackled. Here we read of a computer capable of effecting 25,000 solutions per second, a phenomenal performance. The authors argue that speed and accuracy are to some extent interchangeable; at the outset of their research they decided to pursue methods leading to the highest rate of solutions compatible with an "overall accuracy" of about 5%, which they say is sufficient for many of the problems of physics and engineering. To this end, we read in a chapter in the second part, they constructed an amplifier consisting of a single stage with a cathode follower and positive feedback to boost the gain. By this simplicity they achieved the half-value bandwidth of 750 kc/s requisite for the high speed, but a gain of only 5,000 at most. Again in the first part they state the necessity of providing the computer with means of reducing the rate of solutions for work requiring more accuracy, such as at the end of the exploratory part of the solution of a many-parameter problem. On their computer provision was made to reduce the rate by a half, but no indication is given of an accompanying increase in amplifier gain. However, in a private communication I learned that the authors were primarily interested in the techniques of using a high, variable rate of solutions, and did not actually employ the higher gains permitted at the lower solution rate and required to give the more accurate solution. This thought-provoking part of the book is concluded with a general description of the experimental computer installation at King's College, London, on which the research was carried out.

The second part is devoted to the elements of the ultra-high-speed computer. Here the authors explore in detail linear computing elements, multipliers and function generators; sampling and function storage, control units and multi-dimensional and stereoscopic displays; measurement, power supplies and finally the overall performance of components, a chapter being given to each . They give, of course, the details of their own equipment, but at the same time mention other possible systems in detail. Here is a wealth of information for any designer of a high-speed analogue computer. Of particular interest to the potential user is the chapter on multi-dimensional display. By rotating the axes of co-ordinates and varying the brightness they have been able to produce perspectives of three-dimensional figures on the cathode-ray tube, and have even been able to give unambiguous displays of figures of higher dimension. Stereoscopic display as a three-dimensional alternative is also described. There are many photographs illustrating these effects and also showing solutions of problems to which I will refer later; they show admirably that at the high repetition rate one is able to obtain a simultaneous display of a family of solutions.

In the last part the authors consider the applications of ultra-high speed computer to the solution of ordinary linear and non-linear differential equations, partial differential equations, integral and other functional equations. After a discussion on the possible procedures for solving the most general ordinary differential equation and the accuracy of solutions, the authors go on to explain in detail, by means of examples from theoretical physics, the solution of the initial-value or "marching" problem and the boundary value or "jury" problem. Photographs of three-dimensional displays illustrate the ease and rapidity with which one can obtain the eigenfunctions in the latter, and also a portrayal of the behaviour of the solution at values of the parameter approaching an eigenvalue.

The chapter on non-linear differential equations is most interesting. Indeed the authors rightly say "we now turn by way of illustration to a multi-parameter problem which calls on the full repertoire of the high-speed computer," and again ". . . to convey something of the flavour of problem solving along these new lines, so that potential users with similar problems in quite different contexts may be encouraged to make their own adaptations." The illustration is made by means of the example of the class of equations

$$\frac{dy}{dx} = \lambda \frac{y(x - \mu y + \nu) + \rho}{x(y - \mu' x + \nu') + \rho'}$$

members of which occur in certain branches of physics. An analysis of the equation in an appendix shows the three types of singularity in the (x, y) plane associated with the equation: saddle point, focal point and nodal point. In numerous photographs of families of solutions generated by the automatic variation of the initial condition or a parameter, these singularities show up astonishingly clearly. It is plainly evident that here is a powerful method of discovering solutions of complex non-linear equations quickly, and gaining an understanding of them readily. Indeed, as far as I know, it is the only practicable way of achieving this.

A chapter on partial differential equations treats of the solution by the serial and parallel methods of parabolic, elliptic and hyperbolic equations, and investigates the stability of the processes together with the influence on the solution of component tolerances. The stability investigated is that of the computer process and not that of the iterative process *per se*; thus the serial methods do badly in the appraisal, because of the build up of positive exponentials in the successive stages of the computation, due to the presence of noise and hum, a situation aggravated by the reduction of the difference-mesh. In the following chapter, however, higher-order difference approximations of the derivatives obtained by means of the Lagrange interpolation formula are developed with a view to exchanging the difficulty of the elimination of instability to that of the suppression of spurious solutions. An appendix to this chapter tabulates the interpolation coefficients.

The final chapters of the last part are devoted to the solution of integral equations. The authors point out that there has been little development in this field; so their exposition is largely exploratory. Of great interest is a scheme they have developed for the solution of Fredholm's integral equation of the second kind. This employs a process of iteration whose rate of convergence is shown to be superior to the well known Neumann sequence. Errors are analysed, and methods are proposed for the solution of equations in which the simple iterative methods would diverge; these are the methods of asymptotic convergence, double expansion and kernal transformation. Finally there is a brief discussion on the analogue computation of general integral transforms and the solution of integro-differential equations.

The concluding chapter of the book gives a discussion on the future of ultra-high speed electronic techniques.

The book is addressed to all in the analogue field that aspire to take part in the exciting developments that are possible. It does not lay down principles of computing, but in reporting new ideas and results throws out a challenge to designers and users to accelerate the rate of advance of the art. There is no spirit of rivalry in it towards those in the digital field; indeed, in the final paragraph of the book the authors quite rightly assert " . . the importance of appreciating the *complementary* character of the two great approaches to computation, the digital and the analogue; and the still more exciting possibilities which now exist for their fruitful combination in future computing devices."

The book is well produced and the price in relation to the contents is reasonable. Most chapters have one or more appendices giving the underlying theory of some subject occurring in the chapter. Ample references to works, not limited to the English language, are listed at the end of each chapter. In addition to the normal index, there is an author index.

There are a few misprints, none of which is serious: also there are ambiguities in one or two of the mathematical statements; but in the attempt to interpret these correctly the reader will gain a fuller understanding of the relevant matter. Presumably to keep the size of the book down many steps in many of the mathematical arguments are omitted, imposing heavy though rewarding labour on the curious reader, as I myself have found out.

L. N. CARLING

An Introduction to Electronic Analogue Computers, by M. G. HARTLEY. 1962; 155 pages. (London: *Methuen and Co. Ltd* 21s.; New York: *John Wiley and Sons Inc.*)

The object of this monograph is to provide an introduction to electronic analogue computation for the reader with some knowledge of electronic circuits. The first two chapters discuss the historical background, including mechanical differential analyzers, and the essential differences between analogue and digital computing techniques.

Chapter 3 introduces the operational amplifier and its use in linear function units for summation, integration, and the simulation of more complicated transfer functions. The effect of amplifier gain on the errors which occur in sign-reversing and scale-changing is mentioned, but the effect of grid current is deferred until chapter 5, and the influence of grid current on the accuracy of integrators is not discussed.

Chapter 4 introduces the complete analogue computer and illustrates the setting-up and scaling of a problem, the introduction of the initial conditions and the avoidance of over loading, taking as example a particular second-order equation.

The design of d.c. operational amplifiers using thermionic valves is discussed in chapter 5, including the problem of zero drift and the use of chopper stabilization. The frequency response required for computing amplifiers is barely mentioned, and some discussion of this topic could well replace the rather extended treatment of the causes of zero drift.

The particular problems of transistor computing amplifiers are treated in chapter 6, where some performance figures of typical amplifiers are given, but no mention is made of the advantages of silicon transistors both for input stages and choppers.

The final chapter surveys auxiliary equipment such as multipliers, non-linear function generators and analogue-to-digital converters. A more important item than any of these, however—the means of extracting the required solution from the analogue computing circuit—is not mentioned. Some information about cathode-ray tube displays, chart recorders, and x-y plotters would surely have been more valuable than some of the matter included, since one or more of these is essential to any computing installation.

Two misprints have been noted; a negative sign is missing in the text of Fig. 3.11.*b*, and the word "assumed" at the end of section 3.5 should presumably be "summed." References for further reading are given at the end of each chapter, but there is no bibliography of more advanced textbooks.

Subject to the omissions mentioned, this monograph would be a suitable text for a final-year student in Electrical Engineering, and for post-graduate students of other disciplines such as Mechanical and Civil Engineering, and Nuclear Reactor Technology. For post-graduate electrical engineering courses it would be a useful introduction to the subject, before tackling the longer standard works.

J. C. CLULEY