

A program to study the effect of random delays on the ability of trains to run to a schedule

By Beryl Kitz and S. Vajda

This paper gives an example of a somewhat unusual application of a computer to a combinatorial problem. The problem which is being dealt with is of a typical Operational Research type and concerns the cumulative effect of delays which trains suffer between stations, if some rules are operative which make departures of some trains dependent on the arrivals of others.

The paper describes the structure of the program and the manner in which the results are exhibited, viz.

by comparing the time table according to which trains should run with time tables according to which they might in fact run, and/or

by constructing statistical histograms of delays at certain selected stations, showing the effect of random delays between stations.

Use has been made of a subroutine generating random numbers, which has been used in other A.R.L. projects as well.

The fundamental combinatorial ideas can, of course, be applied to other problems of similar structure, even though they might be expressed in dissimilar terms.

The problem which we consider in this paper is of a finite combinatorial type, and, like all such problems, it can be solved in a finite number of steps. However, the speed of an electronic computer is indispensable if an answer is to be found within a reasonable time.

We deal with the following problem. Assume that a railway time table is given, containing the arrival and departure times of a number of trains at a succession of stations. At its arrival, a train may be late, and this fact may delay its subsequent departure as well as that of other trains. Thus one delay at one station may have effects on the arrivals and departures at other stations, and we want to investigate the extent of these inter-related effects.

The results can be exhibited in two ways. Firstly, the original time table and the final, i.e. actual time table may be exhibited next to one another. Secondly, we can produce, if it is required, a number of such time tables within the computer, which differ from one another in the use of different random delays between stations, and we print a histogram of the delays experienced at specified stations by specified trains.

Clearly, the results depend on the random numbers used, and on the rules connecting arrivals and departures of various trains. We shall formulate the rules, and use the following terms.

Rules

We distinguish between scheduled times, i.e. those in the time table from which we start, and actual times, i.e. those resulting from the delays and their mutual effects.

If the departure of a train depends on the arrival of another, then we call the former the *main train*, and the latter a *pre-train*. It will be assumed here that no train has more than one pre-train at any given station. If the pre-train carries passengers who might wish to change

to the main-train, then we call it a *meeting train*, and otherwise a *crossing train*. This name is due to the idea that as a rule a crossing train will run in the opposite direction to the main train. Its existence might affect the operation of the main train if the line ahead is a single track, so that the main train cannot leave before the crossing train has arrived. We do not assume here that the crossing train may, in fact, be so much delayed that the main train could proceed to the next station where there is a by-passing facility. Nor do we take any overtaking facilities into account.

The rules determining departures of trains are as follows.

A train must stop for at least c minutes (compulsory stop).

If there is a meeting train at the station, then the main train waits until w minutes (changing time) after the arrival of the meeting train, provided that this is not longer than m minutes (maximum stop) after its own arrival. (If there were a maximum waiting time to be counted from the scheduled arrival of the pre-train, then the program would have to be modified.)

If the pre-train is a crossing train, then the main train must await its arrival and may then depart.

All departure rules are, however, subject to the overriding rule that no train departs before its scheduled departure time.

The actual arrival time of a train at a station is its scheduled time, plus any delay in the departure from the previous station, plus a random non-negative number. We do not assume that a delay in a departure can be made up between one station and the next.

In the examples below we use the values $c=3$, $w=5$, $m=8$.

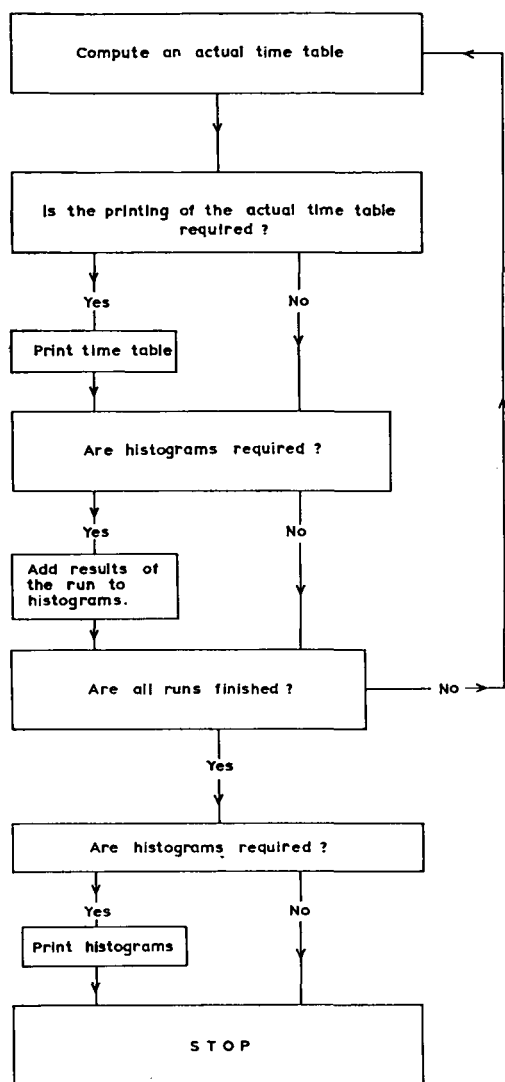


Fig. 1—Overall plan of program

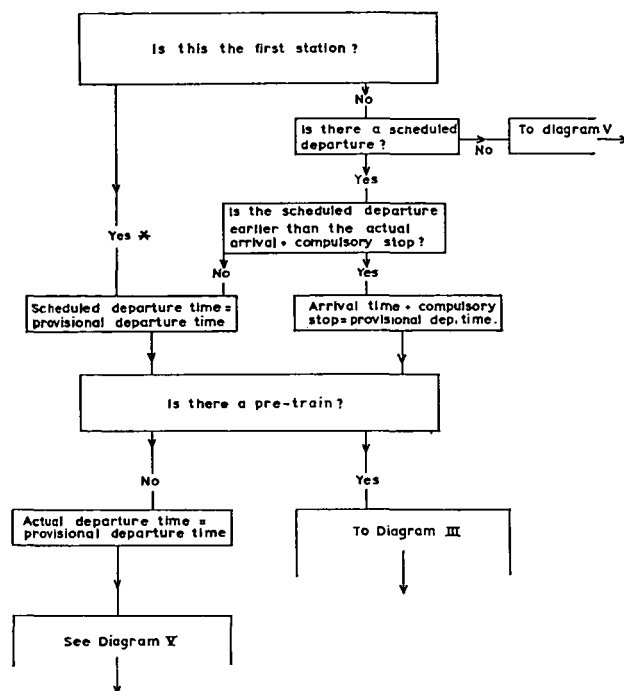
The overall plan of the program is indicated in Fig. 1.

It will be appreciated that even when histograms are required for certain trains at some selected stations only, it will still be necessary to work out the complete time table, because of the relationship between the various times of arrival and of departure.

Logical scheme

We describe now the procedure followed to determine the actual time table. We deal here with the logical scheme, while the computational details will be described in a later section.

To start with, consider a train at a certain station, which is not the terminus of that train. It may be its first station, or one of the later ones. If the latter, then we assume for the time being that its actual arrival there has already been determined. To find the train's departure time, we compare first the scheduled departure time with the actual



* However, see description in text as regards actual computation.

Fig. 2—Departure Time Routine

arrival time plus compulsory stop, and call the later one of these the provisional departure time. This will be the actual departure time, if there is no pre-train at that station. At the first station of any train, the actual departure time will equal the scheduled time. Fig. 2 shows how far we have got.

If there is a pre-train, then we assume, at this stage, that the actual arrival time of the pre-train has also been determined. We must then distinguish between a crossing and a meeting pre-train. When the pre-train is a crossing train, then the actual departure time of the main train is either its provisional departure time, or the arrival time of the crossing train, whichever is later.

When the pre-train is a meeting train, then its actual arrival and the changing time must also be considered. Altogether, the following times will now be relevant: (a) scheduled departure time, (b) actual arrival of main train + compulsory stop, (c) actual arrival of meeting train + changing time, (d) actual arrival of main train + maximum stop. We must be able to deal with a time table where the scheduled departure time is later than the actual arrival time plus the maximum stop. In this case the scheduled time is, of course, overriding. In a formula, the actual departure time equals $\max [a, b]$, $\min [c, d]$. The expression $\max (a, b)$ has already been termed the provisional stop, and the program computes the other two extrema. It is easily seen that the result cannot possibly be smaller than the scheduled departure time. (See Fig. 3).

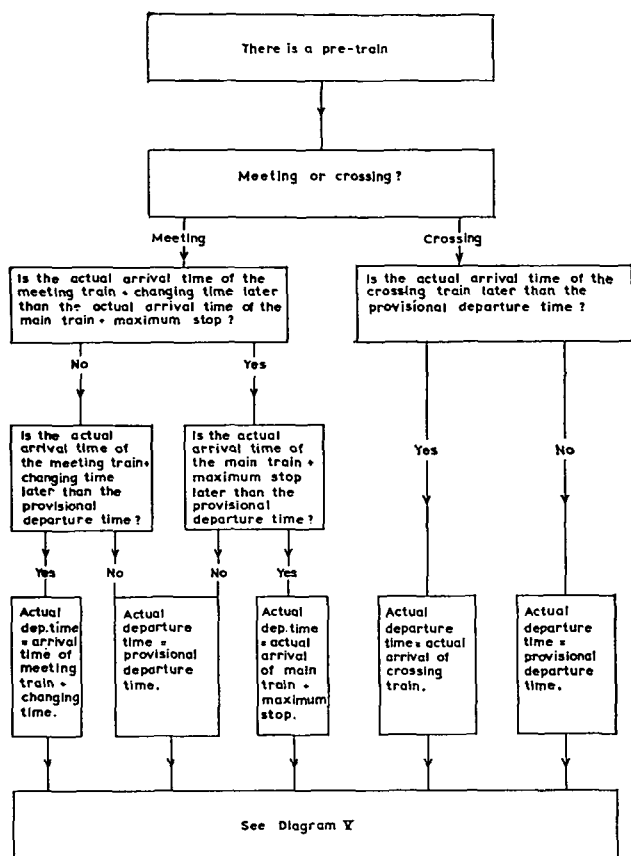


Fig. 3—Departure Time Routine

When describing our Departure Time Routine we assumed that the arrival times of the train considered (called the main train) and of any pre-train we had to take account of, had previously been determined. We shall now describe how to deal with those cases—which frequently arise—where this assumption is invalid.

The routine to be described is called the Train Searching Routine. Starting with the first station of the first train, we search through our list until a station is found from which the actual departure time of the relevant train has not yet been determined. We then see whether the train considered has a pre-train at this station. If not, then we continue with the Departure Time Routine. On the other hand, if there is a pre-train, then we see whether its arrival at this station has been determined. If it has, then we turn once more to the Departure Time Routine to continue with the main train. Otherwise we consider the pre-train as being, from now on, the main train and go back to its first station, to ask whether its departure from there is known. If not, then we repeat the procedure just outlined. Fig. 4 describes the procedure concisely.

We have yet to describe the various ways in which the Departure Time Routine may be left.

When the departure of a given train from a given station has been determined, we store the actual arrival and departure times in a part of the computer not yet used.

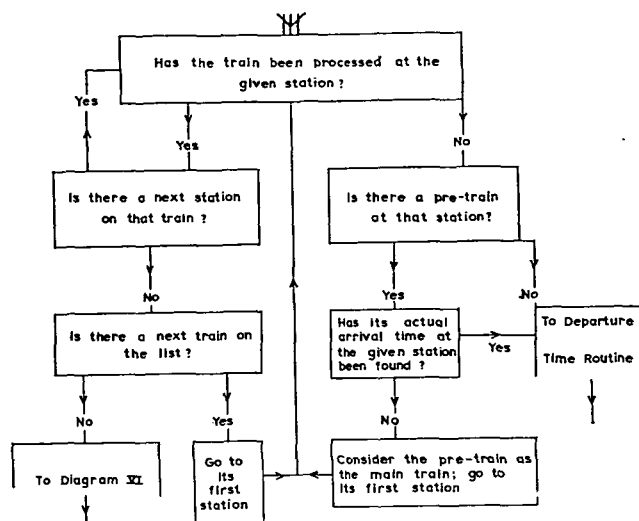


Fig. 4—Train Searching Routine

Having done this, we proceed to compute the actual arrival time at the next station, if there is one, by adding to the scheduled arrival time the departure delay (difference between scheduled and actual departure times) at the present station, and a random or fixed number, indicating a further delay between stations. This part of the Departure Time Routine is by-passed if there is no next station. We then add a marker to the scheduled arrival and departure times, indicating that their actual parallels have been stored.

Having attached the marker, we proceed either to the next station of the train, or if there is no such station, to the first station of the first train on the list, in either case entering the Train Searching Routine (top of Fig. 4, marked M). The plan just described is illustrated in Fig. 5.

In this manner we start again and again at the first station in the list, until eventually all stations of all trains have been processed.

Finally, we remark that the program just described can be used to produce a histogram of the random numbers generated by the Random Number Subroutine used, by restricting our schedule to a pair of stations and registering, in a histogram, the arrival time delays at the second station.

Computational details

We shall now describe the computing aspects of our scheme. The program was written for a Pegasus computer, with paper-tape input, but any modifications required for writing a program on another machine will be obvious.

Two data tapes are required, the first giving the time table information, and the second specifying the trains and stations for which histograms are required, if any. The time table information for each station of every train is given as follows:

Train number (1, 2, . . . 31)—2 decimal digits

overflow from one part of the word to the next does not take place.

The second data tape, referring to the histograms, is read when the input of the scheduled time table has been finished, unless a handswitch is set. It contains a list of train numbers and station numbers for which histograms are required. The maximum number of such histograms is 64.

The computer identifies the pair of train and station numbers and searches for it in the scheduled time table. When the pair has been found, its address is put away to serve later as a modifier for finding the pair in the completed time table of actual arrivals and departures. At the same time the total number of histograms required is counted. Finally, sufficient computer storage is cleared to accommodate the required histograms: this amounts to 64 words for each histogram.

When the number of histograms required—which might be zero—has been determined, and the necessary storage space for them cleared, then the program proceeds to the Train Searching Routine with its subroutine, the Departure Time Routine.

Fundamentally, the computation proceeds as shown in Figs. 2 and 3. The entries in the time table list are tested in sequence to find the first entry which has not been processed. The sign digit is used for this purpose: it is zero to begin with and is changed into one after processing has taken place. When such a train and station have been found, the next test concerns the question of whether a pre-train has to be considered at that station. If there is one, then the word referring to the pre-train at the station under consideration must be found among the train-station-words in the schedule. This involves comparing the pre-train number and current station number with the successive pairs of train and station numbers in the schedule, until the relevant word is found.

If the actual arrival time of the pre-train at the station being considered has not been calculated, then the first station of that pre-train (which now assumes the role of a main train) must be found. For this purpose a note is made of the address of the first station of each train as it is encountered during the search for the pre-train. Thus, when the latter has been found, the address of its first station is immediately available. The question of whether there is a next station of a train is answered by finding whether the next word is zero, and two successive zeros indicate the end of the complete schedule.

In Fig. 2 it is shown that, if we are dealing with a first station, then we may by-pass part of the program, since if there is no arrival, there must be a departure. However, we did not wish to embarrass the program if, by mistake, a train was listed with both arrival and departure as —.—. Thus we considered it to be simpler to let every station have an arrival time, even if, in the case of a first station, only a fictitious one. The arrival time is in such a case constructed as being equal to the departure time less the compulsory stop, and hence this arrival time plus the compulsory stop, equals the scheduled departure, so that the latter equals also the provisional departure time.

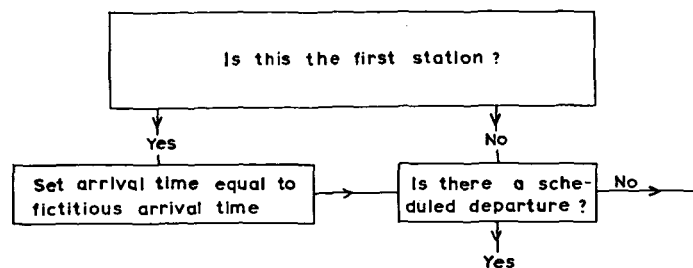


Fig. 6—First-station action

Hence, from a computing point of view, we proceed in the first part of the routine as shown in Fig. 6.

The program is written in such a way that an accumulator receives the departure time (if any) and is changed from scheduled to provisional and to actual departure time, whenever such a change is proved necessary by the result of a test. When it comes to working out the actual arrival time at a station after that previous to it has been processed, then a decision can be made whether a fixed or a random delay should be assumed to occur between the two stations. If a handswitch is set, then the delay is to be a constant, put in through the data tape. We used eight minutes in this case. If the switch is not set, the subroutine generates random numbers.

We made use of a routine which had been written by E. M. L. Beale and which, to begin with, produces random variates uniformly distributed between 0 and 2^{-6} . The sum of 48 such values is then taken to be distributed approximately normally with mean $3/8$ and standard deviation 2^{-5} . This we have adjusted to have mean two minutes and standard deviation four minutes (any other parameters could have been used), and any negative result replaced by zero. A mask restricts, by collation, the random numbers to those digits which refer to the arrival time.

After each time table has been produced and stored, a decision has to be made as to whether the time table should be printed, and/or whether histograms should be constructed. The former must be worked out in any case, but the latter only if their printing is required. The decision depends on two handswitches. If the first of these is set, no time tables are printed, and if the second, no histograms are worked out. If both were set, the computer would stop. If none is set, both time tables and histograms are made ready to be printed. The handswitch controlling the printing of the time tables can be changed at any time during computation. A changing of the other handswitch at a late stage does not have any effect.

Fig. 7 starts at the stage where the actual time table has been worked out. It has one Exit to the Train Searching Routine for the next train, and one to the final STOP. It shows in more detail the procedure already outlined in part of Fig. 1.

There are 64 cells available for each histogram, and each contains the number of delays of length 0, 1–4, 5–8, . . ., 249 or more. Each histogram refers to one train

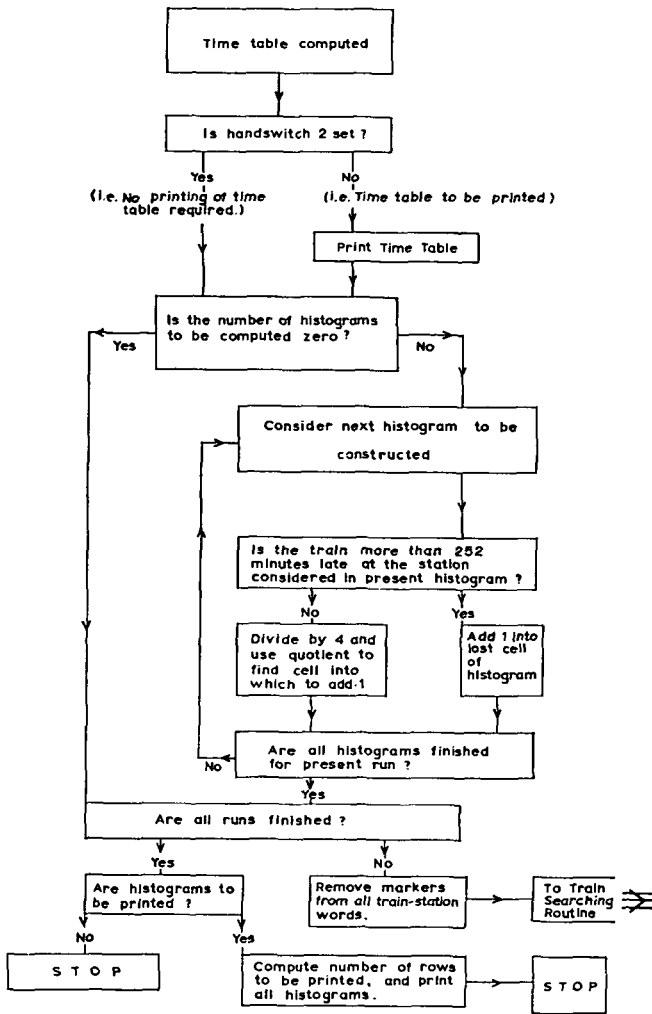


Fig. 7—Trains and their connections

and one of its stations. The cell to which a number is added during the computation of the histogram for each delay at a given train-station pair in a run is determined by adding unity into the last cell if the delay is 252 minutes or more, and otherwise into the cell whose label (0, 1, . . .) is the quotient of the delay plus three minutes, upon division by four, ignoring any remainder. The addition of three minutes has the effect that a zero delay is counted in a cell of its own.

We have arranged that eight histograms are printed side by side and that the contents of the cells are printed in blocks of eight (see Appendix). The sixty-fourth cell of the histogram stores the number of delays of four hours and nine minutes or more. It is likely that in many cases such long delays will never occur. In order, therefore, to avoid printing useless information, we stop printing as

soon as all blocks containing a non-zero entry have been printed.

We must thus calculate the number of cells to be printed before any printing takes place, since we require all histograms to be printed in the same format. This is done in the following way. For each histogram in turn, the entries in the cells, starting with the cell storing the number of zero delays, are totalled, until the sum equals the number of runs carried out (four in our examples) and the number of summands is noted. The number of blocks is readily found from the largest number of summands used for the several histograms.

An example of an input schedule and the resultant output time tables and histograms is given in the Appendix. In this case the number of runs has been set to four and histograms have been requested for ten train-and-station pairs. A diagram showing the connections between the trains is also reproduced below. In this diagram stations are shown as squares, and arrowed lines joining stations with different numbers represent trains. Lines within the station squares represent connections between trains; each line joins the arrival point of a pre-train to the departure point of a main train. A line joining two trains outside a station square indicates a crossing train; here, too, the line joins the arrival point of the pre-train to the departure point of the main train.

Examination of the resultant time tables will show that in only one case out of 32 is a connection missed, namely, in the first run at station 27, train 15 does not wait for train 16. It would appear, therefore, that the connections have been organized satisfactorily. However, from the histograms it is plain that train 16 does not keep to its scheduled time table at either intermediate station 26 or at its final destination 20. It would seem, therefore, that there is a strong case for retiming the arrivals and departure times of this train.

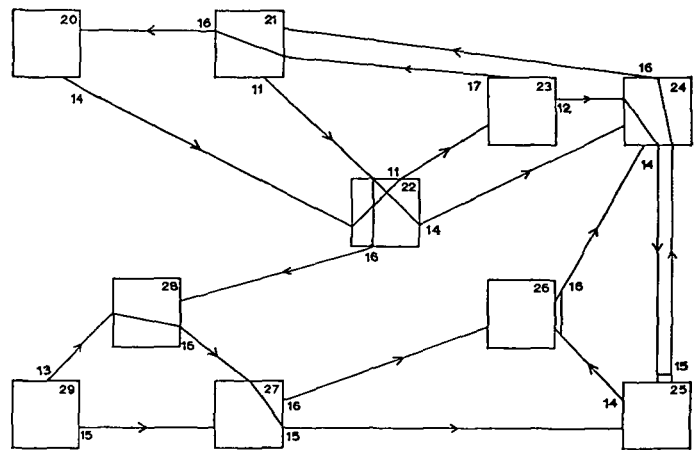


DIAGRAM SHOWING TRAINS AND THEIR CONNECTIONS

Appendix

Initial data time table

I1	21	--. --	23.00	00	0
I1	22	23.19	23.45	I4	0
I1	23	23.59	--. --	00	0
I2	23	--. --	23.20	00	0
I2	24	23.40	--. --	00	0
I4	20	--. --	23.00	00	0
I4	22	23.20	23.24	I1	0
I4	24	23.44	23.53	I2	0
I4	25	00.04	00.08	00	0
I4	26	00.16	--. --	00	0

TRAIN + STATION	I1 22	I1 23	I2 24	I4 26	I3 28	I5 24	I6 20	I7 21
HINS LATE	NO OF TIMES	NO OF TIMES	NO OF TIMES	NO OF TIMES	NO OF TIMES	NO OF TIMES	NO OF TIMES	NO OF TIMES
0	1	2	2	0	1	0	0	1
1-4	2	1	1	1	2	0	0	0
5-8	1	0	1	2	0	1	0	3
9-12	0	1	0	1	1	1	0	0
13-16	0	0	0	0	0	2	3	0
17-20	0	0	0	0	0	0	0	0
21-24	0	0	0	0	0	0	1	0
25-28	0	0	0	0	0	0	0	0

I3	29	--. --	23.30	00	0
I3	28	23.46	--. --	00	0
I5	29	--. --	23.56	00	0
I5	27	00.00+00.04+I6	0	0	0
I5	25	00.08	00.11	I4	I
I5	24	00.22	--. --	00	0

TRAIN + STATION	I4 24	I6 26
HINS LATE	NO OF TIMES	NO OF TIMES
0	0	0
1-4	3	0
5-8	1	3
9-12	0	0
13-16	0	1
17-20	0	0
21-24	0	0
25-28	0	0

Histograms resulting
from time tables
reproduced on next page

I6	22	--. --	23.46	I1	0
I6	28	23.50	23.53	I3	0
I6	27	23.59	00.02+00	0	0
I6	26	00.08	00.16	I4	I
I6	24	00.23	00.26	I5	0
I6	21	00.36	00.39	I7	0
I6	20	00.50	--. --	00	0
I7	23	--. --	00.08	00	0
I7	21	00.35	--. --	00	0

TRAIN + STATION	I1 22	I1 23	I2 24	I4 26	I3 28	I5 24	I6 20	I7 21
HINS LATE	NO OF TIMES	NO OF TIMES	NO OF TIMES	NO OF TIMES	NO OF TIMES	NO OF TIMES	NO OF TIMES	NO OF TIMES
0	0	0	0	0	0	0	0	0
1-4	0	0	0	0	0	0	0	0
5-8	4	4	4	0	4	0	0	4
9-12	0	0	0	0	0	0	0	0
13-16	0	0	0	0	0	0	0	0
17-20	0	0	0	0	0	0	0	0
21-24	0	0	0	0	0	0	0	0
25-28	0	0	0	4	0	4	0	0
29-32	0	0	0	0	0	0	0	0
33-36	0	0	0	0	0	0	0	0
37-40	0	0	0	0	0	0	0	0
41-44	0	0	0	0	0	0	0	0
45-48	0	0	0	0	0	0	0	0
49-52	0	0	0	0	0	0	4	0
53-56	0	0	0	0	0	0	0	0
57-60	0	0	0	0	0	0	0	0

L

Auxiliary tape calling for histograms

I1	22
I1	23
I2	24
I4	26
I3	28
I5	24
I6	20
I7	21
I4	24
I6	26
L	

TRAIN + STATION	I4 24	I6 26
HINS LATE	NO OF TIMES	NO OF TIMES
0	0	0
1-4	0	0
5-8	0	0
9-12	0	0
13-16	4	0
17-20	0	0
21-24	0	4
25-28	0	0
29-32	0	0
33-36	0	0
37-40	0	0
41-44	0	0
45-48	0	0
49-52	0	0
53-56	0	0
57-60	0	0

Histograms obtained using
a fixed delay

Program to study train scheduling
Time tables produced on four successive runs

Run 1				Run 2					
11 21	--.--	23.00	--.--	23.00	11 21	--.--	23.00	--.--	23.00
11 22	23.19	23.45	23.25	23.45	11 22	23.19	23.45	23.19	23.45
11 23	23.59	--.--	00.08+	--.--	11 23	23.59	--.--	23.59	--.--
12 23	--.--	23.20	--.--	23.20	12 23	--.--	23.20	--.--	23.20
12 24	23.40	--.--	23.44	--.--	12 24	23.40	--.--	23.40	--.--
14 20	--.--	23.00	--.--	23.00	14 20	--.--	23.00	--.--	23.00
14 22	23.20	23.24	23.23	23.30	14 22	23.20	23.24	23.20	23.24
14 24	23.44	23.53	23.52	23.55	14 24	23.44	23.53	23.48	23.53
14 25	00.04	00.08	00.11	00.14	14 25	00.04	00.08	00.12	00.15
14 26	00.16	--.--	00.22	--.--	14 26	00.16	--.--	00.28	--.--
13 29	--.--	23.30	--.--	23.30	13 29	--.--	23.30	--.--	23.30
13 28	23.46	--.--	23.55	--.--	13 28	23.46	--.--	23.46	--.--
15 29	--.--	23.56	--.--	23.56	15 29	--.--	23.56	--.--	23.56
15 27	00.00+	00.04+	00.03+	00.11+	15 27	00.00+	00.04+	00.07+	00.10+
15 25	00.08	00.11	00.20	00.23	15 25	00.08	00.11	00.17	00.20
15 24	00.22	--.--	00.36	--.--	15 24	00.22	--.--	00.36	--.--
16 22	--.--	23.46	--.--	23.46	16 22	--.--	23.46	--.--	23.46
16 28	23.50	23.53	23.54	00.00+	16 28	23.50	23.53	23.50	23.53
16 27	23.59	00.02+	00.12+	00.15+	16 27	23.59	00.02+	00.05+	00.08+
16 26	00.08	00.16	00.22	00.25	16 26	00.08	00.16	00.14	00.28
16 24	00.23	00.26	00.34	00.41	16 24	00.23	00.26	00.41	00.44
16 21	00.36	00.39	00.51	00.54	16 21	00.36	00.39	00.57	01.00
16 20	00.50	--.--	01.05	--.--	16 20	00.50	--.--	01.14	--.--
17 23	--.--	00.08	--.--	00.08	17 23	--.--	00.08	--.--	00.08
17 21	00.35	--.--	00.43	--.--	17 21	00.35	--.--	00.41	--.--
Run 3				Run 4					
11 21	--.--	23.00	--.--	23.00	11 21	--.--	23.00	--.--	23.00
11 22	23.19	23.45	23.23	23.45	11 22	23.19	23.45	23.21	23.45
11 23	23.59	--.--	00.03+	--.--	11 23	23.59	--.--	23.59	--.--
12 23	--.--	23.20	--.--	23.20	12 23	--.--	23.20	--.--	23.20
12 24	23.40	--.--	23.40	--.--	12 24	23.40	--.--	23.46	--.--
14 20	--.--	23.00	--.--	23.00	14 20	--.--	23.00	--.--	23.00
14 22	23.20	23.24	23.20	23.28	14 22	23.20	23.24	23.20	23.26
14 24	23.44	23.53	23.48	23.53	14 24	23.44	23.53	23.46	23.53
14 25	00.04	00.08	00.10	00.13	14 25	00.04	00.08	00.08	00.11
14 26	00.16	--.--	00.21	--.--	14 26	00.16	--.--	00.19	--.--
13 29	--.--	23.30	--.--	23.30	13 29	--.--	23.30	--.--	23.30
13 28	23.46	--.--	23.48	--.--	13 28	23.46	--.--	23.48	--.--
15 29	--.--	23.56	--.--	23.56	15 29	--.--	23.56	--.--	23.56
15 27	00.00+	00.04+	00.03+	00.06+	15 27	00.00+	00.04+	00.00+	00.07+
15 25	00.08	00.11	00.10	00.13	15 25	00.08	00.11	00.17	00.20
15 24	00.22	--.--	00.29	--.--	15 24	00.22	--.--	00.33	--.--
16 22	--.--	23.46	--.--	23.46	16 22	--.--	23.46	--.--	23.46
16 28	23.50	23.53	23.50	23.53	16 28	23.50	23.53	23.50	23.53
16 27	23.59	00.02+	23.59	00.02+	16 27	23.59	00.02+	00.02+	00.05+
16 26	00.08	00.16	00.13	00.21	16 26	00.08	00.16	00.16	00.19
16 24	00.23	00.26	00.34	00.37	16 24	00.23	00.26	00.33	00.38
16 21	00.36	00.39	00.47	00.50	16 21	00.36	00.39	00.51	00.54
16 20	00.50	--.--	01.03	--.--	16 20	00.50	--.--	01.06	--.--
17 23	--.--	00.08	--.--	00.08	17 23	--.--	00.08	--.--	00.08
17 21	00.35	--.--	00.41	--.--	17 21	00.35	--.--	00.35	--.--