

approximation depends on the coefficients α , β , γ being such that

$$t^4\gamma < t^3\beta < t^2\alpha. \quad (13)$$

Since the factor $1/x$ is common to all terms, the relative error is independent of frequency, and the inequality (13) is the condition on range of variable over which the polynomial approximation can be used. The relative

errors from two-term or one-term approximation respectively are then of order

$$t_1\beta/\alpha \text{ and } t_1\gamma/\beta.$$

If one knows approximately the values of the coefficients α , β , γ one can choose the bound t_1 of the interval of approximation so that the error resulting for a given order of approximation is admissible.

References

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 CAMPBELL, G. A., and FOSTER, R. A. (1948). *Fourier Integrals for Practical Applications*, New York: Van Nostrand.
 PUCCEL, R. A. (1957). "Network Synthesis for a Prescribed Impulse Response using a Real-Part Approximation," *Journ. App. Phys.*, Vol. 28, p. 124.

Appendix: Use of a digital computer

The Fourier integral can be approximated by a summation of the form

$$F(\omega) = \delta t \sum_{k=1}^n f(k\delta t) \left(\frac{\sin}{\cos} \right) \omega k \delta t \quad (i)$$

where δt is the increment of independent variable and $t = n \delta t$ is the value of independent variable at which the function is truncated. There are Fourier series and Fourier integral programs for most computers which are used for scientific work, but usually with the increments of ω in arithmetic progression. But in deriving spectra, which would be plotted on logarithmic scales, one often wants to cover several decades of ω with frequencies in geometric progression. The main feature of the program which has been constructed to

do this is that instead of working in fixed increments of angle ωt and finding the values of $f(t)$ at the corresponding values of t , one takes the ordinates at given values of t and applies the appropriate increment of angle in passing from one ordinate to the next:

$$F(\omega) = \Delta t \sum_{k=1}^n f_k(t) \left(\frac{\sin}{\cos} \right) k \delta \theta \quad (ii)$$

where $\delta \theta = \omega \Delta t$ and Δt is the interval between ordinates. It is then never necessary to interpolate between given values of the true function.

The program is in FORTRAN and the author wishes to thank IBM United Kingdom Ltd. for their unstinted assistance with this work, and for a grant under their Research Endowment scheme of machine time for the testing and use of the program.

Book Review

Mathematical Methods for the Study of Automatic Control Systems. By V. I. ZUBOV, 1962; 327 pages. (Oxford: Pergamon Press, 84s.)

This book contains a comprehensive account of qualitative methods for the investigation of control systems. Most of it is concerned with the mathematics of the subject, leaving only the final chapter for a discussion of the part of computers in this work.

The theory of non-linear control systems is developed in a logical manner. Starting in Chapter 1 we have definitions of stability in the sense of Lyapunov, and of technical stability. This is followed by general theorems on the stability of motion and on regions of stability. Chapter 2 goes on to discuss transient processes and stability of linear systems. This chapter ends with the description of a method of determining the stability of a linear system without the use of the characteristic polynomial. From the matrix of the coefficients of the system another matrix is formed. For stability it is necessary that the eigenvalues of this matrix all lie in the unit circle which can be determined by forming high powers of the matrix.

In Chapter 3 conditions are obtained for the stability of non-linear systems including the cases of technical stability and systems with time delays. Chapter 4 is concerned with the behaviour of systems near to singularities, and Chapter 5 with the effects of random perturbations on the transient

solutions of systems. The next two chapters deal with cases where the linear approximations used so far no longer work. Chapter 6 is for cases where non-linear approximations have to be used to investigate stability, while Chapter 7 discusses self-oscillatory systems.

The final chapter is divided between analogue and digital computers. First there is a brief description of the principles of operation of the computers, and this is followed by a description of the machines available in Russia when the book was originally published in 1959. After this the use of the method described in Chapter 2 for the synthesis and analysis of control systems with the aid of a digital computer is presented, and finally the application of digital and hybrid computers for automatic optimization of systems is described.

This book is a thorough monograph on the study of non-linear control systems, a subject that has been largely developed in Russia. In the preface to the English edition, the author expresses the wish that the publication of the book will assist other workers to develop further methods of studying control systems. I feel certain that this hope will be fulfilled.

This is a very well produced book and the translation on the whole reaches a similar high standard. Occasionally however the translation reads strangely and words are not rendered in the usual form, e.g. for *autonomous* there appears at several points the word *autonomic*.
 J. HOUSTOUN.