The data for an evaluation are the numerical values of the orbital parameters A, α, etc. The first stage of the calculation is the evaluation of all the auxiliary functions, it not being thought worthwhile to program the selection only of those required for the particular integral. Similarly no attempt was made to carry over the values of any of the auxiliary functions from one integral to another.

The successive terms of the multinomial were then evaluated and their sum accumulated. At this stage a check on the accuracy of the evaluation was carried out. All arithmetic was in the floating-point mode with a binary exponent and a nominal precision of 38 significant binary digits. The maximum exponent over all terms and over all partial sums was recorded. The excess of this over the exponent of the final result is a measure of the differencing error incurred. Numerous evaluations have suggested that this error does not amount to more than two or three binary digits.

In the evaluation of each term $C\prod_j x_j^{u_j}$ the array u was scanned and zero entries ignored. The remaining factors were formed by using the digits of the binary representation of mod(u_j) to select contributions from the sequence obtained by repeatedly squaring x_j or $1/x_j$. Each term was evaluated independently of the rest. Were it not for the limited rapid-access storage available on the machines used, it might have been worthwhile preserving some information from one term to the next. Thus the sequence x_j, x_j^2, x_j^4, ... might have been preserved. For $u_j > 0$, $x_j^{u_j}$ would have been calculated as a product of selections from this sequence as before; for $u_j < 0$ then $1/x_j^{-u_j}$ would have been calculated. The terms of the sequence would best be calculated when first required; the number of terms currently available could be stored and the sequence could readily be extended as needed.

It is thought that the speed of evaluation could be significantly increased on suitable machines in this way. There are, however, further improvements which are theoretically possible and which would be very rewarding if means could be devised for implementing them automatically. The simplest example is the insertion of brackets in the expression for a polynomial prior to evaluating it by nested multiplication thus:

$$ax^3 + bx^2 + cx + d = ((ax + b)x + c)x + d.$$

Still more efficient means of evaluating polynomials are discussed by Knuth (1962). The present paper shows how general functions of several variables may be reduced to multinomials, albeit with some loss of efficiency due to unexploited dependency among the auxiliary functions. Even in this limited field, however, there appears to be no theoretical treatment of the evaluation problem. Any proposal, to be useful in the present context, must be capable of evaluating efficiently an expression whose precise form is not known to the programmer. This is, of course, a much greater problem than that posed by the evaluation of any particular expression, however complex.

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