Theory and Design of Digital Machines, by THOMAS C. BARTEE, IRWIN L. LEBOW, IRVING S. REED. 1962; 324 pages. (London: *McGraw-Hill Publishing Co. Ltd.*, £4. 9s. 0d.)

Many different disciplines are involved in the design of digital computers and "an unlikely mixture of engineers, logicians, mathematicians and philosophers have contributed to the present-day state of the digital art."

Each regards the computer in the light of his own specialization. Thus, the engineer views the design problem as one of circuit design and of interconnecting systems components of different types to provide the required systems function. In this he tends, sometimes, to miss the subtleties of a more abstract mathematical approach although often creating ingenious solutions, bordering on genius, in the process.

On the other hand, the abstract approach of the mathematician, or logician can frequently fail to appreciate a lucrative area for investigation, which a familiarity with practical design problems would show.

The problem is basically one of communications between the many groups engaged in computer research. The overwhelming need is for a common language, which would enrich the activities of each group, facilitate the integration of the many approaches to computer design and lead to a much-needed broadening of the overall theory.

The book under review makes some progress towards this goal. Aimed at the practising engineer and student it combines the more traditional specializations of machine design and switching theory, and presents an integrated, systematic mathematical approach to computer design, based on a symbolic representation of transfer operations between registers.

Thus, a digital computer unit is regarded as a set of independent and dependent registers interacting through transfers specified by a control unit. A realistic computing system is then treated as a set of interacting computing units.

The design process is considered in three distinct phases:

(*a*) systems design, which sketches in the general configurations of the machine and specifies the general class of hardware to be used;
(*b*) structural design, which describes the system in terms of transfer relations;
(*c*) logic design, which realizes the transfer relations by means of Boolean equations.

Accordingly the book divides naturally into three basic sections:

(*a*) the foundations of machine design, in which the basic components, such as the register and combinational network, are introduced and defined, together with a mathematical notation which conveniently describes their operation;
(*b*) a theoretical treatment of the mathematical foundations of switching theory based on Boolean rings and fields, and systematic methods of minimizing Boolean expressions using the well-known Quine-McCluskey method and introducing more recent algorithms;
(*c*) various aspects of machine design, introducing the basic sequential networks and showing how, with the addition of a simple control unit, an elementary computer can be evolved. This concept is expanded through the addition of the arithmetic unit and a program unit to a simple general-purpose computer. The full design method is then developed and illustrated with two special-purpose computer designs, the first an automatic radar detection and processing computer and the second a digital differential analyser.

A final chapter discusses more advanced, theoretical subjects such as minimal-state machines, Turing machines, probabilistic machines and finite-state automata. Number representation systems are dealt with in an Appendix.

The treatment is limited to logical design subjects and makes only passing reference to programming, electronic-circuit designs and numerical methods.

The book is very well written in a lucid style although, at times, rather simple concepts are unnecessarily complicated in the interests of mathematical rigour. Questions follow each chapter to illustrate and amplify the material in the text. Copious references are provided after each chapter for further study, although there seems little point in the needless repetition this involves.

The mathematical rigour of the treatment makes heavy going for the engineer familiar with more direct methods of design, and the authors do not always show convincing reason why the rigour is necessary. Thus the Quine-McCluskey technique is presented as a manual method for switching-circuit minimization, whereas its elegance and popularity is largely due to the ease with which it can be programmed for a computer-based design procedure. It is the general adoption of computer techniques for computer design that makes mathematical methods such as those presented by the authors so important, and makes the book itself worthwhile reading for all interested in computer design.

K. L. SMITH

A Survey of Mathematical Logic, by HAO WANG, 1963; 651 pages. (Amsterdam: *North-Holland Publishing Company*, 120s.)

Although this is primarily a collection of papers, it is more than just a collection, for some of the papers have been amplified and the collection welded into a whole by the introduction of supplementary matter. Wang writes clearly and persuasively and has interesting and important things to say about a wide range of topics.

The book is divided into five parts. The first of these starts with a chapter on the axiomatic method, and contains informal accounts of Gödel's famous incompleteness theorems. This is followed by a reprint of the paper, "Eighty years of foundational studies," which first appeared in the Festschrift of Bernays (*Dialectica* 1958), and a paper on the axiomatization of arithmetic (from the *Journal of Symbolic Logic*, 1957). The section concludes with an excellent discussion of the concept of computability, which contains an account of Mucnik and Friedberg's solution of Post's problem in which they showed that there is a recursively enumerable set B (i.e. a recursive function f) which is not recursive (i.e. $Ex\{y = f(x)\}$ is not a recursive relation), and a recursively enumerable set A which is not recursive in B.

The second part of the book is devoted to the theory of calculating machines and reproduces papers from the *Journal of the Association for Computing Machinery*, *the Zeitschrift f. math. Logik*, and the *IBM Journal of Research and Development*, on universal machines, the logic of automation, the proof by a machine of 400 theorems in *Principia Mathematica*, and circuit synthesis by solving sequential Boolean equations (i.e. Boolean equations with an additional "time" difference operator).

The third part opens with a general discussion of predicate logic, of the first and higher orders, and the arithmetization of metamathematics, with emphasis on the consistency problem, and Gödel's theorems. Skolem's non-standard model of arithmetic (a set of functions of ordinal greater than ω which play the part of the natural numbers in a formalized arithmetic), and Ackermann's consistency proof by transfinite induction are clearly explained.

In the fourth and fifth parts on impredicative and predicative set theories we move into the field in which Wang has done his best-known work. The fourth part opens with a very informative survey of a variety of axiomatizations of set theory, the Zermelo theory, that of Paul Bernays, Russell's theory, those of Quine, and Gödel's set theory. This is followed by a reprint of a paper from the *Mathematische Annalen* (1953) on relationships between number theory and set theory, and a summary of the contents of Wang's Harvard (1948) doctoral dissertation, much of which has been published in the *Journal of Symbolic Logic*; another paper from that Journal, on the formalization of mathematics, discusses the problems of consistency and adequacy. The book ends with a paper from the *Zeitschrift f. math. Logik* (1959) on ordinal numbers and predicative set theory, which discusses Herbrand arithmetic definitions and Herbrand partial recursive definitions.

Published simultaneously by the North Holland Publishing Company and The Science Press, Peking, this book was printed and bound in China. The printing is good and clear, but the paper is not of the quality to which the Dutch printers have accustomed us in this series. The quality of the contents is happily beyond question.

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THE COMPUTER JOURNAL

Published Quarterly by

The British Computer Society, Finsbury Court, Finsbury Pavement, LONDON, E.C.2, England.

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