Acknowledgements

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References

BELLMAN, R., and DREYFUS, S. (1963). *Applied Dynamic Programming*, Oxford University Press.
STONE, R., BROWN, J. A. C., and others (1962). *A Project for Growth*. Chapman and Hall, London, Parts 1, 2, 3.

Correction to "Eigenvectors of the successive over-relaxation process, and its combination with Chebyshev semi-iteration", by G. J. Tee

The above paper was published in this *Journal*, Vol. 6, No. 3, pp. 250–263 (October 1963). Part of §8 is now known to be in error. After the sentence containing equation (8.7), the remainder of §8 should be replaced by the following:

"Let $w_1, \ldots, w_\beta$ be a set of linearly independent eigenvectors of $\boldsymbol{H}$ with non-zero eigenvalue η. The eigenvectors of $\boldsymbol{H}$ given (cf. (8.4)) by $v_i = \boldsymbol{S}^{-1}w_i$ $(i = 1, \ldots \beta)$ are linearly independent; for if

$$c_1v_1 + \ldots + c_\beta v_\beta = 0, \tag{8.8}$$

then pre-multiplication by $\boldsymbol{S}$ would give

$$c_1w_1 + \ldots + c_\beta w_\beta = 0, \tag{8.9}$$

and hence $c_1 = \ldots = c_\beta = 0$, since the w_i are linearly independent. Let the eigenvalue $\lambda = \eta^{-\frac{1}{2}}\zeta$ of $\boldsymbol{K}$ have multiplicity μ, so that we can construct μ linearly independent eigenvectors $v'_1, \ldots, v'_\mu$. It follows that β is not greater than μ. We may show in a similar manner that the vectors $w'_i = Sv'_i$ $(i = 1, \ldots, \mu)$ are linearly independent eigenvectors of $\boldsymbol{H}$ with eigenvalue η, and hence the maximum possible value of β is exactly μ.

Provided that equation (4.9) does not hold, the multiplicity of η is μ; but if (4.9) does hold, i.e. if

$$\omega = \frac{2}{1 \pm \sqrt{(1 - \lambda^2)}} \tag{8.10}$$

then $\eta = \omega - 1$ is an eigenvalue of $\boldsymbol{H}$ with multiplicity 2μ. We conclude that $\boldsymbol{H}$ has a complete set of linearly independent eigenvectors unless $\eta = \omega - 1$ is an eigenvalue, in which case the corresponding eigenvectors span a space of dimensionality equal (if $\eta \neq 0$) to half the multiplicity of η. Furthermore, $\omega - 1$ is an eigenvalue of $\boldsymbol{H}$ for not more than n distinct values of ω, given by (8.10), where λ is any eigenvalue of $\boldsymbol{K}$. In particular, the error operator for optimized S.O.R. $(\omega = \omega_0)$ does not have a complete set of eigenvectors, since ω_0 is given by (8.10) with $\lambda = \max. \lambda_i$."

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Errata

"Elementary divisors of the Liebmann process," by G. A. Miles, K. L. Stewart, and G. J. Tee, *The Computer Journal*, Vol. 6, No. 4, pp. 352–355 (January 1964).

We regret that certain misprints occurred in this article. The following corrections should be noted:

(1) P. 353, line before (2.12), replace "m_i" by "λ_{m_i}".

(2) P. 354, third line after (3.13), replace "$\frac{1}{2}(n - m)$" by "$\eta^{\frac{1}{2}(n-m)}$".

(3) P. 355, third line after (3.18), replace "$\eta^{1/2(n-m+v+a)}$" by "$\eta^{\frac{1}{2}(n-m+v+a)}$".