where $B = (-b)^{N+1}$. Using nested multiplication the evaluation of x_1 from this equation requires $2N(M + A)$ operations and the evaluation of B, which is comparable with the method of Evans and Forrington. In fact, in terms of programming effort and accuracy it is better to find x_1 as in the latter method when $0 < b^2 < 1$. However, when $b^2 = 1$, the evaluation of x_1 from equation (9) takes only $N(M + A)$ operations.

If theie are two or more sets of equations (1) with a given matrix A_N, equation (8) can be written in the form

$$x_1 = \sum_{i=1}^{N} \frac{(-b)^{i-1}(1 + b^2 + \ldots + b^{2N-2i})d_i}{1 + b^2 + \ldots + b^{2N}} \quad \text{when } 0 < b^2 < 1. \qquad (11)$$

The values of b^{2i} can be found by successive multiplication of b^{2i-2} by b^2, and $1 + b^2 + \ldots + b^{2N}$ can be found by repeated addition of these values. If the expression $(-b)^{i-1}(1 + b^2 + \ldots + b^{2N-2i})$ is denoted by a_i, then a_i can be found from the recurrence relations

$$\left.\begin{aligned} a_{2j} &= -b(a_{2j-1} - b^{2N-2j}) \\ a_{2j+1} &= -ba_{2j} - b^{2N-2j} \end{aligned}\right\} (j = 1, \ldots, [N/2]) \qquad (12)$$

where $a_1 = 1 + b^2 + \ldots + b^{2N-2}$.

The calculation of all the a_i takes $2N(M + A)$ operations, so that if there are P different right-hand sides with a given A_N, the complete solution will take $(2 + 3P)N(M + A)$ operations, compared with $4PN(M + A)$ opeiations using the method of Evans and Forrington.

Now consider the case where $a'^2 < 4b'^2$ and $a' > 0$. If the equations (1) are normalized so that b' is replaced by -1 the equation for D_i is now

$$D_i = aD_{i-1} - D_{i-2} \qquad (i = 2, \ldots, N) \qquad (13)$$

where $D_1 = a$ and $D_0 = 1$, and the equation for x_1 is

$$x_1 = \sum_{i=1}^{N} \frac{D_{N-i}d_i}{D_N}. \qquad (14)$$

Since $|a| < 2$, the recurrence relation (13) provides a reasonably stable means of calculating the D_i (N.P.L., 1961), needing approximately $N(M + A)$ operations. Using these values of D_i in equation (14) involves a further $N(M + A)$ operations to find x_1.

It is not possible to use the method of Evans and Forrington to find the remaining x_i, since it is unstable when $a^2 < 4b^2$. However, the normalized equations (1) can be written as

$$x_{i+1} = ax_i - x_{i-1} - d_i \qquad (i = 1, \ldots, N - 1) \qquad (15)$$

where $x_0 = 0$, and the stability properties of equation (15) are the same as those of equation (13). The evaluation of the remaining x_i from equation (15) requires approximately $N(M + 2A)$ operations.

If A_N appears with only one right-hand side the complete solution will require $N(3M + 4A)$ operations, and if A_N appears with P right-hand sides the complete solution will require $N(2P + 1)$ multiplications and $N(3P + 1)$ additions.

Acknowledgement

The author would like to thank the referee for his comments on the stability of the calculations described in a previous version of this paper. As a result of these comments the paper has been almost completely rewritten.

References

EVANS, D. J., and FORRINGTON, C. V. D. (1963). "Note on the solution of certain tri-diagonal systems of linear equations," *The Computer Journal*, Vol. 5, p. 327.

N.P.L. (1961). *Modern Computing Methods*, 2nd edition, London: H.M.S.O.

Correspondence

To the Editor,

The Computer Journal.

Sir,

With reference to my article "The numerical solution of second-order differential equations not containing the first derivative explicitly" (*The Computer Journal*, Vol. 6, p. 368), I have been informed that formula (D) given there has also been given by Albrecht (1955). I am grateful to Dr. Albrecht for drawing my attention to this.

Yours faithfully,

R. E. SCRATON.

Northampton College of Advanced Technology,
St. John Street, London E.C.1.

Reference

ALBRECHT, J. (1955). "Beiträge zum Runge–Kutta-Verfahren," *Z. angew. Math. Mech.*, Vol. 35, p. 100.