

## References

- COURANT, R., and HILBERT, D. (1953). *Methods of mathematical physics*, Vol. I, New York and London: Interscience.
- FOX, L. (1964). *Introduction to Numerical Linear Algebra*, Oxford: Clarendon Press.
- FOX, L. (Ed.) (1962). *Numerical solution of ordinary and partial differential equations*, Oxford: Pergamon.
- FOX, L., and GOODWIN, E. T. (1953). "The numerical solution of non-singular linear integral equations," *Phil. Trans. Roy. Soc. A*, Vol. 241, p. 501.
- PHILLIPS, D. L. (1962). "A technique for the numerical solution of certain integral equations of the first kind," *J. Ass. Comp. Mach.*, Vol. 9, p. 84.
- TRICOMI, F. G. (1957). *Integral equations*, New York and London: Interscience.

## Book review: Linear programming

*Linear Programming and Extensions*, by GEORGE B. DANTZIG, 1963; 625 pages. (Princeton: University Press; London: Oxford University Press, 92s.)

This long-awaited book by Professor Dantzig will certainly come to be regarded as one of the classic textbooks on linear programming. I can highly recommend it to all interested in the theory of linear programming and its more useful extensions, particularly to those interested in the economic interpretation of linear programming models, and also for use as a library reference book. The treatment of the subject matter is mathematically complete, and explained in sufficient detail to make it as readable as possible. This approach, together with the wealth of material which the book contains, explains its great size. It gives almost comprehensive coverage to the main developments in the subject until the end of 1960.

Just how many years ago it was that I first heard that Dr. Dantzig was writing a book on linear programming I now forget. But as the years passed and it failed to appear, "Dantzig's book" became almost a joke. Now the preface reveals the great organization that went into its preparation. No less than 24 people, most of them well known in linear programming circles, including five professors and nine doctors, are thanked for contributing to the writing of various sections, and a further nine people for helping with the layout, proof-reading and indexing. The result is a book of authority and of technical excellence.

Naturally, with so many hands contributing, the style is not entirely uniform. But this variety adds a certain spice. Some sections in which the ideas are conveyed in story form such as "The Scheme of the Ambitious Industrialist" make particularly good reading, whilst in other chapters the ideas of several papers are rather loosely strung together. An occasional remark reveals the long period of gestation, such as that in the second chapter about developments in the ten years since 1947.

The chapter on the origins and early influences on linear programming is of great interest. But Professor Dantzig is too modest to allow the reader to recognize the full impact which the team under his leadership at the Rand Corporation had on the rapid development of this subject in the nineteen fifties.

Professor Dantzig believes in the usefulness of approaching his subject in many ways in order to gain as many insights into it as possible. The simplex method is therefore explained not only in terms of linear equations and inequalities, but also in terms of matrix algebra, with two distinct geometrical interpretations, economic interpretations and an equivalence with matrix games.

Likewise he covers the transportation problem in a variety

of manners: in terms of its own techniques, in terms of the simplex method and in terms of networks and trees. Various special cases, extensions and generalizations of transportation problems and network problems are also discussed.

Extensions to linear programming are treated in five chapters. The first explains the valuable concept of Wolfe's generalized programming scheme. There follows an excellent chapter on the decomposition of linear programs. This includes a short play in which Staff, who hates details, and his economist friend, F. M. Dalks, use the decomposition principle to get Sub to buy the right number of tankers. It also includes a section on the use of the decomposition principle for central planning without complete information at the centre.

A chapter on convex programming includes mention of separable and quadratic programming. There is a chapter on uncertainty, at the start of which the reader is warned that the treatment is necessarily fragmentary as few problems have been solved in this area. Lastly there is a chapter on integer programming which includes a good description of Gomory's method of integer forms but is otherwise mainly a survey of problems which can be put into integer programming form.

In places a number of exercises are suggested to the reader, and many chapters end with a list of problems, a few of which are marked as unsolved.

In his opening sentence Professor Dantzig states firmly that the final test of a theory is its capacity to solve the problems which originated it. Therefore it is surprising to find that he pays so little attention to the implementation of linear programming techniques on digital computers. Whilst there is an elaborate discussion on perturbation techniques which is of great theoretical interest, there is virtually no mention of the perturbation through rounding errors which occurs in all numerical work, nor of the effect which such errors can have on the course of the calculations.

In the detailed iterative procedure given for the two-phase simplex method there is no mention of the tolerance on zero which must be allowed because of rounding errors, and without which the procedure can fail to terminate correctly. The flow diagram given of the simplex method is by no means the most suitable for direct implementation on a computer. The "product form of the inverse" algorithm, which is nowadays most commonly used for large-scale linear programming calculations, receives such a brief mention on page 200 that it is not to be found in either the list of contents or the index. For these reasons the book is not recommended as a handbook for programming linear programming calculations for computers. A good book for that purpose has yet to be written.

MARTIN FIELDHOUSE.