An analogue computer simulation of a Cowper Stove

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The hot blast stove is a counterflow regenerator operating under conditions of variable mass flow. In consequence the solution of the corresponding differential equations for dynamic equilibrium makes extreme demands on computing facilities. An analogue circuit has been developed which involves few simplifying assumptions. Results have been compared with a simplified digital solution for a suitably restricted case.

The blast furnace must be supplied with hot air at constant flow rate and constant temperature. By means of regenerative heat exchangers, called *Hot Blast Stoves* or *Cowper Stoves*, the blast for the furnace is preheated by the hot waste products of burnt blast-furnace top gas.

The cycle of operation of the stove consists of a heating period and a cooling period. In the heating period, blast furnace gas of calorific value 90–100 B.Th.U./cu.ft. is burnt, usually at constant rate, but with an excess air ratio which is varied so as to limit the surface temperature of brickwork at the entry end of the stove. The hot products of combustion are passed down through a matrix of ceramic material, called "chequerwork," in which the heat is then stored. At the end of this period the gas is shut off.

In the cooling period, the heat is regenerated from the chequerwork to the air for the blast furnace, which is blown upwards through the heat-storing mass. Constancy of blast temperature is achieved by varying the proportion of the total air delivered to the furnace which passes through the stove. To provide a continuous supply of a blast, at least two stoves are required. Typical operating times are one hour on blast (cooling period), and, with the more common 3-stove installation, two hours on gas (heating period). A typical modern installation might be required to furnish 100,000 s.c.f.m. of air at 1000°C.

The behaviour of such a system is not difficult to formulate in the form of differential equations, but obtaining the solution of the equations corresponding to a repetitive cyclic state is a formidable problem unless drastic simplifying assumptions are made. Until the appearance of machines such as the Atlas digital computer, simulation could only be effected at prohibitively high cost or by extensive simplification. Analogue computers appeared to offer more flexibility with some loss of numerical accuracy, and they have been used by the research departments of the Jones & Laughlin Steel Corporation (Meyer, Simcic, Ceckler and Lander, 1960) and the United States Steel Corporation (Schuerger and Agarwal, 1961). The present work was based on the extensive facilities incorporated in the analogue computer of the International Research & Development Company at Newcastle upon Tyne, and represents a part of a combined analogue and digital computer attack on the problem.

The mathematical model

The differential equations considered in this work as representing the thermal behaviour of a Cowper Stove are as follows:

1. Heat transfer within the chequer heat-storing mass

$$\frac{\partial T}{\partial \theta} = \alpha \frac{\partial^2 T}{\partial x^2} \tag{1}$$

(longitudinal conductivity in y — direction ignored).

2. Heat transfer between the solid surface and the gas flowing through the channels of the chequerwork

$$hA\,(T_0 - t) = WSL\frac{\partial t}{\partial y} \tag{2}$$

3. The boundary conditions relating the two equations are:

$$h(T_0 - t) = \lambda\left(\frac{\partial T}{\partial x}\right)_{x\,=\,0 \text{ and } x\,=\,d} \tag{3}$$

Further, $$0 = \left(\frac{\partial T}{\partial x}\right)_{x\,=\,\frac{1}{2}d} \tag{4}$$

4. For heating period $W = f\,(R)$; initially $R = R_0$, subsequently $R = f\,(\theta)$ such that $T_0\,(y = 0) =$ constant. In the cooling period W varies with time and, at any instant, satisfies the relationship $W = W_B(t_B - t_0)/(t_8 - t_0)$.

Notation
$y =$ direction of gas flow (feet)
$x =$ direction into chequer wall, perpendicular to y (feet)
$\theta =$ time (hours)

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$T = T(x, y, \theta)$ = solid temperature (°F)
$t = t(y, \theta)$ = gas temperature (°F)
$T_0 = T_0(y, \theta)$ = surface solid temperature (°F)
α = chequer thermal diffusivity (ft^2/hr)
h = surface heat transfer coefficient (B.t.u./ft^2/hr/°F)
A = heating surface area of chequerwork (ft^2)
W = mass flow rate of the gas (lb/hr)
S = specific heat of the gas (B.t.u./lb/°F)
L = length of Cowper Stove (y — direction) (feet)
λ = chequer thermal conductivity (B.t.u./ft/hr/°F)
d = chequer wall thickness (x — direction) (feet)
R = excess air ratio (ft^3/ft^3 gas)
W_B = total mass flow rate of air delivered (lb/hr)
t_B = blast temperature for furnace (°F)
t_8 = air temperature at stove outlet (°F) (at $y = L$)
t_0 = air temperature at stove entrance (°F) (at $y = 0$).

Difference representation of the differential equations

The hyperbolic and parabolic differential equations involve derivatives with respect to θ, time and x, y, distance. The integration facilities in an analogue computer can only be used to integrate with respect to one dimension, usually as in this case θ time. The other derivatives must therefore be expressed in difference form. (See Fig. 1a.)

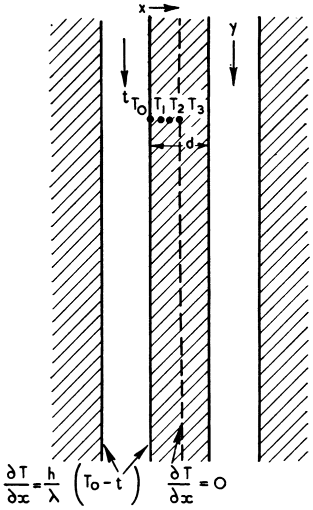


Fig. 1(a)—Illustrating application of diffusivity equation

However, it is first necessary to re-write equation (1) in terms of computer time, τ (seconds) instead of a real time θ (hours). If $\tau = \beta\,\theta$ where, in this problem, β was set equal to 12, that is 12 seconds computer time equivalent to 1 hour of real time, then equation (1) becomes:

$$\frac{\partial T}{\partial \tau} = \frac{\alpha}{\beta}\frac{\partial^2 T}{\partial x^2}. \tag{1a}$$

This equation (1a) is represented in difference form. The solid temperature was calculated at the surface of the solid ($x = 0$, $r = 0$), the middle of the chequer wall ($x = \frac{1}{2}d$, $r = 3$) and at two intermediate equally spaced points ($r = 1$, $r = 2$). The distance between the points was $x = d/6$. In general, at any point r, the equation (1a) becomes:

$$\frac{\partial T_r}{\partial \tau} = \frac{\alpha}{\beta\,(\Delta x)^2}(T_{r+1} - 2T_r + T_{r-1}). \tag{5}$$

In the middle of the chequer wall, the boundary condition (4) is incorporated, that is:

$$\frac{\partial T_3}{\partial \tau} = \frac{2\alpha}{\beta(\Delta x)^2}(T_2 - T_3). \tag{6}$$

At the surface of the solid, the difference representation must incorporate the boundary condition (3), and becomes:

$$\frac{\partial T_0}{\partial \tau} = \frac{2\alpha}{\beta(\Delta x)^2}(T_1 - T_0) + \frac{2h\alpha}{\beta\lambda(\Delta x)}(t - T_0). \tag{7}$$

The truncation error associated with all the approximations for the first and second derivatives is of the order $(\Delta x)^2$.

In the direction y, along the length L of the stove, the solid and gas temperatures were calculated in the analogue model at the two entrances to the chequerwork ($y = 0$, $s = 0$) and ($y = L$, $s = 8$), and at seven intermediate levels equally spaced at a distance $\Delta y = L/8$. (See Fig. 1*b*.)

The entrance gas temperature t_0 is known at all times during the cycle.

Using the trapezoidal quadrature formula, the gas temperature at a level $s + 1$ can be calculated using the equation:

$$t_{s+1} = t_s + \frac{\Delta y}{2}\left\{\left(\frac{\partial t}{\partial y}\right)_{s+1} + \left(\frac{\partial t}{\partial y}\right)_s\right\}. \tag{8}$$

The truncation error associated with this finite-difference representation is:

$$-\left(\frac{\Delta y}{12}\right)^3\left(\frac{\partial^3 t}{\partial y^3}\right) + \ldots$$

This means that, provided an analogue circuit can be devised to compute the derivative $(\partial t/\partial y)_s$ at each level $s = 0, 1, 2, \ldots, 8$, the gas temperature can also be computed. Such a set of nine circuits was included in the analogue-computer model each of which computed, at the level s,

$$\left(\frac{\partial t}{\partial y}\right) = \frac{hA}{WSL}(T_0 - t).$$

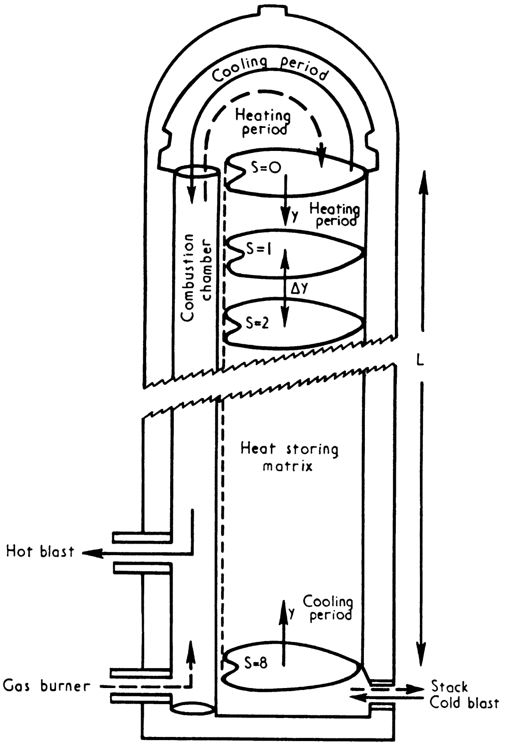


Fig. 1(b).—Schematic drawing of Cowper Stove showing levels for which gas and solid temperatures were calculated

Analogue representation of the unsteady state heat conduction in the walls of the chequerwork matrix

The purpose of this section of the circuit is to generate values of T_0, the surface temperature, from the heat transfer rate $h(t - T_0)$ which is provided from the section of the circuit dealing with gas/solid heat transfer.

Preliminary attempts were made to solve equations (5)–(7) using integrating amplifiers as indirect analogues. Stability was difficult to achieve so recourse was made to the use of a direct network analogue in the feedback of an operational amplifier (Fig. 2).

The difference equations have been set out as equations (5), (6) and (7). In the direct network analogue, use is made of the similarity in form between these difference equations in heat flow in the solid, and the equation for the voltage distribution in a capacitor-resistor network in the feedback loop of an operational amplifier. This is illustrated by the differential equation representing a general part of the network (Fig. 3a).

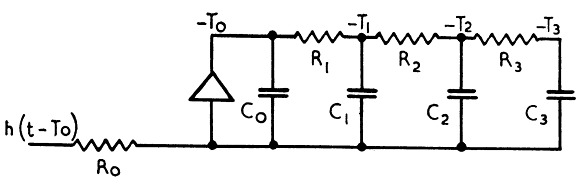


Fig. 2.—Network analogue of unsteady state conduction

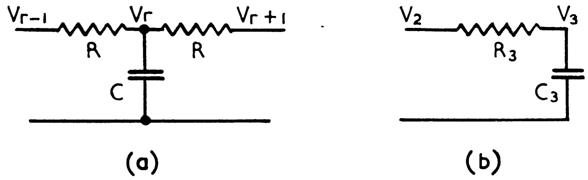


Fig. 3.—Network elements (*a*) and (*b*)

Consideration of Kirchhoff's laws leads to the equation

$$\frac{\partial V_r}{\partial \tau} = \frac{1}{CR}(V_{r-1} - 2V_r + V_{r+1}).$$

Notation V_r = potential at the rth capacitor in the network (volts)
C = capacitance (microfarads)
R = resistance (megohms)
τ = computer time (seconds).

The corresponding difference equation (5) is

$$\frac{\partial T_r}{\partial \tau} = \frac{\alpha}{\beta(\Delta x)^2}(T_{r+1} - 2T_{r-1}).$$

The scaling factor relating voltage to temperature used in this analogue was 25°F/volt. The network equation then becomes, on substitution of $V = T/25$,

$$\frac{\partial T_r}{\partial \tau} = \frac{1}{CR}(T_{r+1} - 2T_r + T_{r-1}).$$

It will be seen that the scaling factor 1/25 does not appear in this equation. It follows that the network equations can all be expressed in terms of temperature T.

If the network is to simulate unsteady-state heat conduction in the chequer mass, it is necessary that

$$CR = \beta\,(\Delta x)^2/\alpha.$$

In the network, it is convenient to choose the value of C_1 and C_2 at each stage to be $1\mu F$. Hence:

$$R_1 = R_2 = R_3 = \beta\,(\Delta x)^2/\alpha \text{ megohms.}$$

In the middle of the chequer wall, the equation becomes:

$$\frac{\partial T_3}{\partial \tau} = \frac{2\alpha}{\beta(\Delta x)^2}(T_2 - T_3).$$

The corresponding analogue representation is shown in Fig. 3b with

$$\frac{\partial T_3}{\partial \tau} = \frac{1}{C_3 R_3}(T_2 - T_3).$$

It follows that $C_3R_3 = \beta(\Delta x)^2/2\alpha$. However, the resistance R_3 is fixed for the previous difference equation for $r = 2$, that is

$$R_3 = \beta\,(\Delta x)^2/\alpha \text{ megohms.}$$

It immediately follows for $r = 3$, that $C_3 = \frac{1}{2}\mu F$.

At the surface boundary, where $r = 0$, the difference equation is

$$\frac{\partial T_0}{\partial \tau} = \frac{2\alpha}{\beta(\Delta x)^2}(T_1 - T_0) + \frac{2\alpha}{\beta\lambda(\Delta x)}h(t - T_0).$$

The corresponding equation for the analogue representation (see Fig. 2) is

$$\frac{\partial T_0}{\partial \tau} = \frac{1}{C_0 R_1}(T_1 - T_0) + \frac{1}{C_0 R_0}h(t - T_0).$$

Hence $C_0 R_1 = \beta(\Delta x)^2/2\alpha$, and, as at the boundary in the middle of the chequer wall, $C_0 = \frac{1}{2}\mu F$ since $R_1 = \beta(\Delta x)^2/\alpha$.

Similarly $C_0 R_0 = \beta\lambda(\Delta x)/2\alpha$. Since $C_0 = \frac{1}{2}\mu F$, the expression for the input resistor R_0 is:

$$R_0 = \beta\lambda(\Delta x)/\alpha \text{ megohms.}$$

Nine such circuits, corresponding to the levels $s = 0, 1, \ldots, 8$, were set up, and it was possible to introduce different components in each to allow for variable geometry and physical properties of bricks where these were not constant throughout the stove setting. As the approximate average brick temperatures were known for each level, appropriate values for the thermal properties could be used. Components R_1, R_2 and R_3 with values of the order of 300 KΩ were made up of fixed resistors in series with 100 kΩ variable resistors. R_0, about 1 MΩ was a fixed resistor preceded by a variable attenuator.

Analogue simulation of heat transfer between gas and brick surface (Fig. 4)

These circuits at each level $s = 0, 1, \ldots, 8$, had inputs corresponding to the gas/solid temperature differences $(T_0 - t)$, and were required to evaluate the rate of heat transfer $h(t - T_0)$ to feed the corresponding diffusivity circuits of Fig. 2 described above, and also the values

$$\left(\frac{\Delta y}{2} \cdot \frac{\partial t}{\partial y}\right)$$

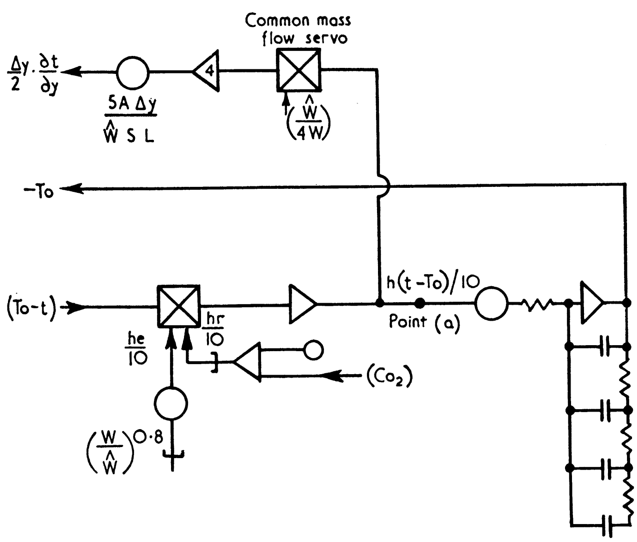


Fig. 4.—Analogue of heat transfer between gas and solid

involved in the evaluation of the gas temperatures t_0 to t_8.

The system involves multiplication of $(T_0 - t)$ by the sum of the convective and radiative heat transfer coefficients h_c and h_r (see below). An individual servo multiplier is used with twin input resistors to the associated amplifier. The multiplier is followed by sign reversal to give $h(t - T_0)$ for input to the diffusivity circuit.

Since division by the mass flow W is performed at each of the nine levels this is effected by a multiple-gang servo multiplier with a factor $\hat{W}/(4W)$ where $\hat{W}$ is an arbitrary maximum flow rate. The factor 4 is recovered in a sign-reversing stage, and an attenuating potentiometer introduces the remaining constants to provide

$$\left(\frac{\Delta y}{2} \cdot \frac{\partial t}{\partial y}\right).$$

The trapezoidal ladderwork

At each level s, of the Cowper Stove model, the quantity

$$\frac{\Delta y}{2}\left(\frac{\partial t}{\partial y}\right)_s$$

was computed by the circuits just described. The nine levels were interlinked by the "trapezoidal ladderwork" of Fig. 5 which is an analogue representation of the previously discussed trapezoidal quadrature formula.

It would have been possible to compute the gas temperature at each level throughout the simulation by direct application of this trapezoidal rule, namely:

$$t_{s+1} = t_s + \frac{\Delta y}{2}\left\{\left(\frac{\partial t}{\partial y}\right)_{s+1} + \left(\frac{\partial y}{\partial y}\right)_s\right\}.$$

However, the evaluation of these temperatures was not of particular interest, except at the exit to the stove. Of more importance was the computation of the temperature difference $(T_0 - t)$ between the surface solid temperature and the gas temperature, since the rate of heat transfer at each level was directly proportional to this difference.

For scaling reasons only $\frac{1}{2}t_0$, where t_0 was the entrance gas temperature, was presented.

The summing amplifiers used at each level to evaluate the temperature difference $(T_0 - t)$ are labelled L_s, that is $L0$, $L1$, $L2$, ..., $L8$. In addition it was necessary to evaluate the gas temperature explicitly at levels $s = 2$, $s = 4$ and $s = 6$ and at the exit where $s = 8$. The values of t_2, t_4, t_6 and t_8 were computed by adders labelled $L9$, $L10$, $L11$ and $L12$, respectively. This temperature difference $(T_0 - t)$ was computed in three different ways at different levels in the stove. At each level, however, the surface solid temperature, computed by the analogue representation of unsteady heat conduction at that level, always comprised one of the input voltages to the adder labelled L_s $(s = 0, 1, \ldots, 8)$.

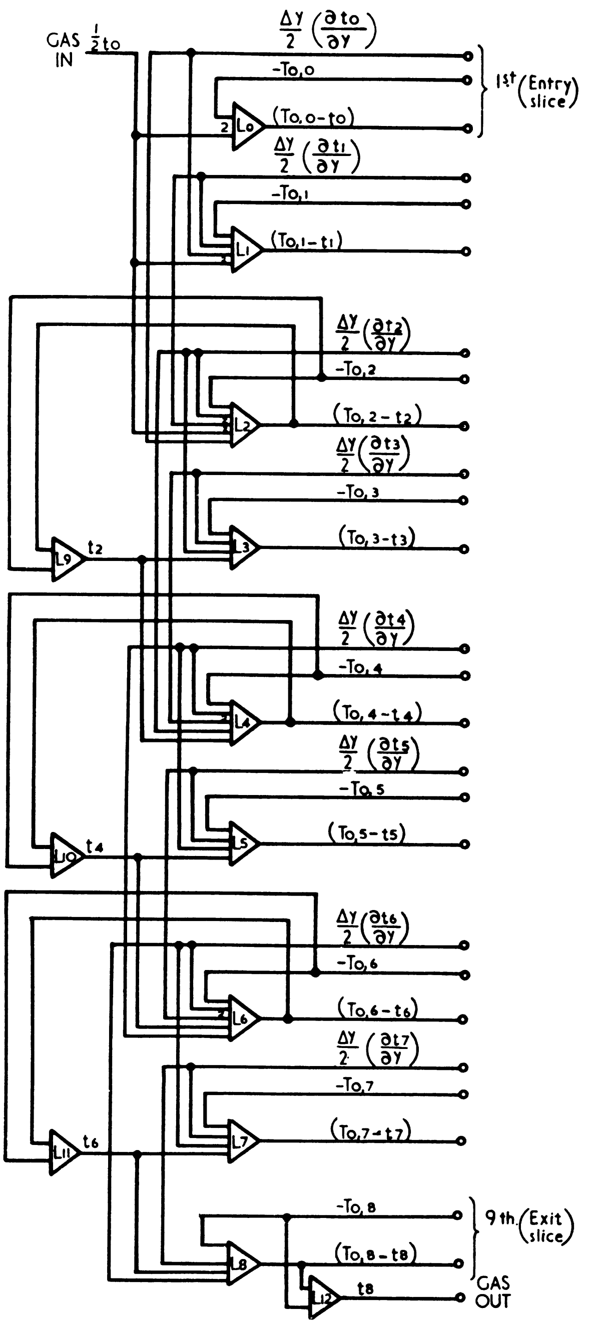


Fig. 5.—Trapezoidal ladderwork for progressive generation of gas temperatures

At level $s = 0$

The input voltages to adder $L0$ corresponded to $-T_{0,0}$ and $\frac{1}{2}\, t_0$ (input with an amplification of 2), yielding the difference $(T_{0,0} - t_0)$.

At levels $s = 1$, 3, 5 *and* 7

Employing the trapezoidal rule, the gas temperature t_s was evaluated in the L_s adder as

$$t_s = t_{s-1} + \frac{\Delta y}{2}\left\{\left(\frac{\partial t}{\partial y}\right)_s + \left(\frac{\partial t}{\partial y}\right)_{s-1}\right\}.$$

The adder inverted the sign of t together with the sign of the other input $-T_0$ yielding the difference $(T_0 - t)$.

Al levels $s = 2$, 4, 6 *and* 8

As above, the gas temperature t_s was evaluated implicitly in the adder L_s using the trapezoidal rule,

$$t_s = t_{s-1} + \frac{\Delta y}{2}\left\{\left(\frac{\partial t}{\partial y}\right)_s + \left(\frac{\partial t}{\partial y}\right)_{s-1}\right\}.$$

However, instead of computing t_{s-1} the trapezoidal representation of this temperature, also namely

$$t_{s-1} = t_{s-2} + \frac{\Delta y}{2}\left\{\left(\frac{\partial t}{\partial y}\right)_{s-1} + \left(\frac{\partial t}{\partial y}\right)_{s-2}\right\}$$

was used, and the expression for t_s became

$$t_s = t_{s-2} + \frac{\Delta y}{2}\left\{\left(\frac{\partial t}{\partial y}\right)_s + 2\left(\frac{\partial t}{\partial y}\right)_{s-1} + \left(\frac{\partial t}{\partial y}\right)_{s-2}\right\}.$$

Including the surface solid temperature $-T_0$ there were thus five inputs to the adder L_s which computed the difference $(T_0 - t)$.

For levels $s = 3$, 5 and 7 it was necessary to evaluate t_{s-1} explicitly employing adders $L9$, $L10$, and $L11$. The exit temperature t_8 was also computed explicitly in the adder $L12$. In order to perform this calculation the voltages corresponding to $(T_0 - t)$ and to $-T_0$ were added together in the summing amplifier, which upon the incumbent sign reversal yielded explicitly the required temperature t_s.

Reversal

The heat transfer and diffusivity circuits shown in Fig. 4 relate to defined positions in the regenerator because the values T_0 are in the nature of a historical time integral and, in addition, many of the potentiometer settings relate specifically to average temperature levels in the stove.

In the ladder system, however, the value t_0 must refer to the input to the chequerwork, the hot end of the system during the heating period and the cold end during cooling. Means must therefore be provided for interchanging the connections between the ladder and the heat transfer computing circuits during the changeover period of the system. Details of the relay system and associated timing devices are given below.

Gas flow conditions

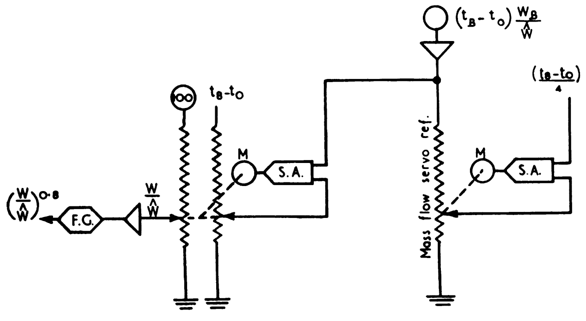
(a) *Cooling period*

During the blast period the inlet temperature t_0 is constant but the flow is partitioned between regenerator and bypass to give a fixed mixed air temperature, t_B.

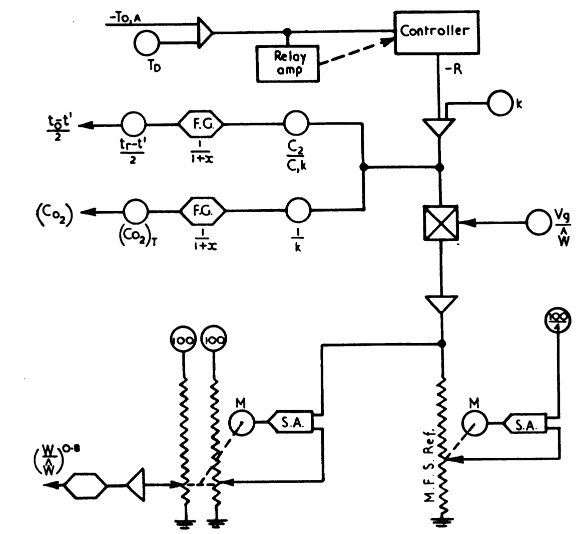
Flow through the stove W is related to total flow to the blast furnace W_B by the relation

$$W(t_8 - t_0) = W_B(t_B - t_0) = \text{constant}.$$

In Fig. 6(a) the multiplying function $\hat{W}/(4W)$ is generated on a multigang servo multiplier by offering $\frac{1}{4}(t_8 - t_0)$



(a) COOLING CYCLE



(b) HEATING CYCLE

Fig. 6.—Input conditions (*a*) **cooling cycle**
(*b*) **heating cycle**

S.A. = multiplier servoamplifier; F.G. = function generator

to the servo amplifier and $(t_B - t_0)\, W_B/\hat{W}$ to the end of the reference potentiometer arc. Simultaneously $W/\hat{W}$ was evaluated on another multiplier and converted to $(W/\hat{W})^{0 \cdot 8}$ in a function generator for use in supplying the convective heat transfer coefficient.

(b) *Heating period*

Gas flow, temperature and CO_2 content are all determined by the air/gas ratio which, after an initial period, is varied to prevent the hottest brickwork surface, $T_{0,A}$ exceeding an arbitrary limiting temperature T_D.

In Fig. 6(b) the ratio of excess air to fuel gas is provided by a 3-term controller analogue operated by the difference $(T_{0,A} - T_D)$ but held to an initial condition state R_0 until $T_{0,A}$ first reaches the value T_D.

Total gas flow $W = V_g(k + R)$ where V_g = volume of fuel gas (constant)

k = volume of combustion products with theoretical air per unit of fuel gas.

By referring the value of W to $\hat{W}$ and $\frac{1}{4}\hat{W}$ the desired factors $\hat{W}/(4W)$ and $W/\hat{W}$ and hence $(W/\hat{W})^{0 \cdot 8}$ are obtained.

The inlet gas temperature is obtained from a heat balance

$$(t_T - t^1)\, k\, C_1 = (t_0 - t^1)\,(k\, C_1 + R\, C_2)$$

where t^1 = ambient temperature
t_T = adiabatic flame temperature for stoichiometric combustion
k = volume of theoretical products per unit volume fuel gas
C_1 = specific heat of same
t_0 = mixed gas inlet temperature
C_2 = specific heat of air.

This can be transformed to give

$$(t_0 - t^1) = (t_T - t^1)\frac{1}{1 + R\left(\frac{C_2}{C_1 k}\right)}$$

and evaluated with a $1/(1 + x)$ function generator and two potentiometers.

The CO_2 content of the mixture of theoretical combustion products and excess air can be obtained rather more simply:

$$(CO_2) = (CO_2)_T\left(\frac{1}{1 + R/k}\right).$$

Reversal

Relays were provided to convert Fig. 6(a) to Fig. 6(b). As reversal of stoves requires about 5 minutes (= 1 second computer time) this otherwise dead period was usefully employed in allowing the 20-gang mass flow servo $(\hat{W}/(4W))$ to reach the appropriate position for the next half-cycle.

Heat transfer coefficients

The convective heat transfer coefficient required was of the form $h_c = h_0 W^{0 \cdot 8} t^{0 \cdot 2}$. Simulation of the $t^{0 \cdot 2}$ term was not included but different constant-time means were taken for the 9 circuits of Fig. 4. A common source $(W/\hat{W})^{0 \cdot 8}$ from Fig. 6(a) or (b) was attenuated by the factor $h_c t^{0 \cdot 2}\,(\hat{W})^{0 \cdot 8}$ and presented to one of the inputs of the h multiplier of Fig. 4.

The rate of heat transfer due to radiation is

$$\sigma\left(\frac{\epsilon_s + 1}{2}\right)(\epsilon g t^4 - \beta g T^4)$$

and may be evaluated with the aid of the Hottel charts. A close approximation is given by

$$(a + b\,(CO_2))\quad (t - T)$$

over a useful temperature range within the overall temperature limits of the present simulation. On the basis of the (CO_2) values given by Fig. 6(b) three functions of the form $a + b$ (CO_2) appropriate to three different mean temperature levels were generated. During the heating period (only) the (second) inputs of the h multipliers of Fig. 4 were fed with the function appropriate to the mean temperature level in the slice.

Timing and relay sequencing

A uniselector driving from a 1 second clock was employed to define the periods

- A on blast (cooling)
- Y changeover (one second = 5 minutes real time)
- G on gas (heating)
- X changeover (one second).

During periods X and Y all inputs to the diffusivity circuits were interrupted (at (a), Fig. 4). At the end of periods A and G the ladder amplifiers $L0$ to $L8$ were protected from overflow due to transients by inserting supplementary feed-back resistors of 100 $k\Omega$. After a time delay of 0.3 seconds, derived from a slugged relay, the control circuits Figs. 6(a) and (b) and the connections between ladder and heat transfer circuits were changed over. The suppressor resistors of $L0$ to $L8$ were then disconnected in that order by a fast relay chain. At the end of the X and Y periods the diffusivity circuit inputs were restored.

Example calculation

It is not the purpose of this paper to discuss the thermal calculations performed using this analogue simulation. These will be set out in a separate paper. However, an example calculation is now presented since it demonstrates that this analogue method of regenerator calculation is comparable with other methods.

Simplification of the differential equations (1) and (2) together with the boundary conditions (3) and (4) is possible by the introduction of an overall heat transfer coefficient $\bar{h}$, from the mean solid temperature T_m, where

$$T_m = \frac{1}{d}\int_0^d T(x)dx,$$

$$\bar{h}\,(T_m - t) = (T_0 - t) \text{ and } 1/\bar{h} = 1/h + d\,\phi/6\,\lambda.$$

It can be shown that in this case, the equations become

$$\frac{\partial T_m}{\partial \theta} = \frac{\bar{h}A}{MC}(t - T_m) \qquad (5)$$

$$\frac{\partial t}{\partial y} = \frac{\bar{h}A}{WSL}(T_m - t). \qquad (6)$$

Notation
- $\bar{h}$ — overall heat transfer coefficient (B.t.u./ft^2/hr/°F)
- ϕ — correction factor for regenerator reversals
- M — mass of chequerwork in regenerator (lb)
- C — specific heat of chequerwork (B.t.u./lb °F).

A numerical method for solving these equations (5) and (6) has been set out in a paper by Willmott (1964) and programmed for the Ferranti Pegasus computer.

The method assumes

(i) that the flow rate in each period is constant with time

(ii) that the inlet gas temperatures in each period are constant with time

(iii) that the heat transfer coefficients in each period do not vary over each period and over the height of the stove.

An initial calculation was performed by the analogue simulation with the control system adjusted and potentiometers set so that these assumptions were satisfied. A comparison was then made between the surface solid temperatures and gas temperatures computed by the analogue simulation and the digital computer program.

Data employed

Thickness of chequer walls	(d)	1.62 (inches)
Chequer specific heat	(c)	0.32 (B.t.u./lb/°F)
Chequer thermal diffusivity	(α)	0.02 (ft^2/hr)
Mass of chequers	(M)	540 (tons)
Surface heat transfer coefficient in both periods	(h)	6.507 (B.t.u./ft^2/hr/°F)
Heating surface area	(A)	127,303 (ft^2)
Flow rate of gas in each period	(W)	70,000 s.c.f.m.
Density of gas	(ρ)	0.08071 (lb/ft^3)
Specific heat of gas	(S)	0.27 (B.t.u./lb/°F)
Chequer thermal conductivity	(λ)	0.76 (B.t.u./ft/hr/°F)
Hot inlet gas temperature	($t_{h,i}$)	2100 (°F)
Cold inlet gas temperature	($t_{c,i}$)	200 (°F)

Heating Period = Cooling Period = 1.5 (hours).

Both the analogue and digital computer programs employed the trapezoidal quadrature formula to integrate up the height of the stove. The temperatures were computed by each method at the entrance and exit to the stove chequerwork, and at seven other levels equally spaced in the height of the stove.

In the analogue method, surface solid temperatures were computed, whereas the digital computer program evaluated mean solid temperature. However, the surface solid temperature can be calculated from the mean solid temperature and gas temperature, using the relation

$$T_0 = t + \bar{h}\,(T_m - t)/h.$$

For this comparison, it is convenient to define the reduced length Λ and reduced period Π, each the same for both periods of the cycle in this case.

$$\Lambda = \frac{\bar{h}A}{WS} = 7.601$$

$$\Pi = \frac{\bar{h}AP}{MC} = 2.676.$$

In Fig. 7 the surface solid temperature at levels 0, 2, 4, 6 and 8, as calculated by both the analogue and digital computer simulations, and denoted by S_0, S_2, S_6 and S_8, are represented as functions of time. In Fig. 8 the air temperatures in the cooling period and the waste gas temperatures in the heating period at levels 2, 4, and 6 are represented as functions of time. The air temperatures are denoted by A_2, A_4 and A_6 and the waste gas temperatures by G_2, G_4 and G_6.

Truncation errors

The truncation errors associated with the trapezoidal ladderwork representation of equation (2) have been investigated previously in a paper by Willmott (1964). They are a function of the departure from linearity of the variations of temperature with time, which in turn are dependent upon the values of Π and Λ. (Hausen (1950).) For the conditions studied with the analogue it was shown that the number of slices employed was adequate.

Since the variation of solid temperature was linear with time except near the gas entrances, the temperature profile within the chequers is almost parabolic, so that the difference replacement of equation (1) was adequate for the number of steps employed.

Operation

When the analogue machine was switched on all values of solid temperature T_0 were zero, and with repeated cycling the system approached a state of dynamic equilibrium. In order to judge when successive cycles were similar the outlet gas temperatures t_8 were integrated over the appropriate half cycles and monitored. In general this required 10 to 15 complete cycles, typically a total run of 10 minutes.

Single point recorders were used to show all surface solid temperatures T_0, exit gas temperature t_8 and various parameters such as W, R and $(T_{0,A} - T_D)$.

Although static checks on the analogue showed that it was possible, by exercising great care in setting up, to achieve 0.1% accuracy, the overall error of the system in dynamic equilibrium appeared to be about 0.5%. This may be ascribed in part to the complexity of the circuit and in part to errors associated with the recording system.

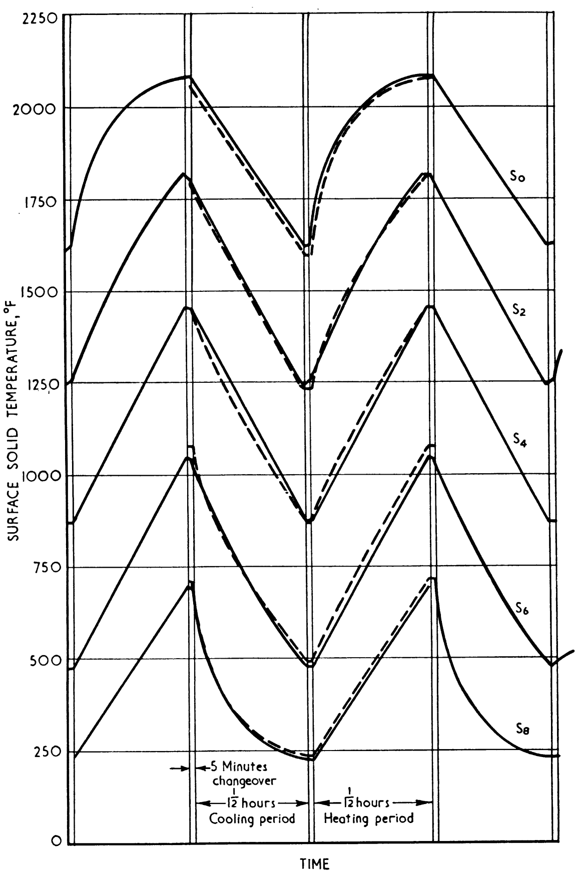


Fig. 7.—Surface solid temperatures calculated by the analogue (–) and by digital simulation (—)

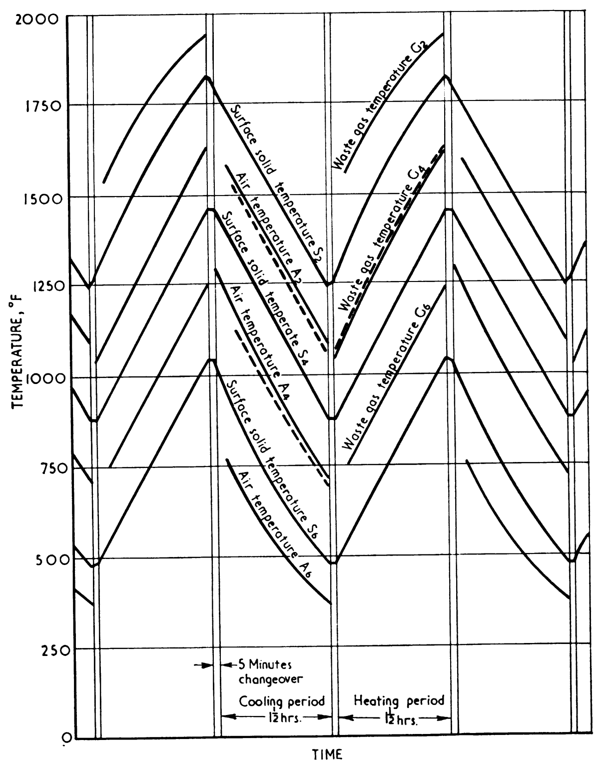


Fig. 8.—Fluid temperatures calculated by the analogue (–) and by digital simulation (—)

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Book Review

Automatic Control & Computer Engineering—Volume 2. Edited by V. V. SOLODOVNIKOV, 1963; 331 pages. (Oxford: *Pergamon Press Ltd.*, 100s.)

The Computer Engineering of the title is presumably to be found in Volume 1, which your reviewer has not read, since there is no vestige of it in the present volume. The book is a collection of papers on automatic control presented at a conference of the Russian instrument-making industry, published as a book in Russia in 1959, and translated into English by the Pergamon Press.

The papers form a *pot pourri* of control theory, each highly theoretical and each highly specialized in its own field. They constitute an important contribution to the classical literature of control, and their publication in English is of value both for its general interest as an indication of developments in the U.S.S.R., and for the individual interest of particular papers to specialist control engineers. It is not for general reading, nor of general interest to computer specialists.

Two out of the nine papers, however, present the theory of pulsed (and sampled) systems, and are in consequence of great interest to those, theoreticians particularly, specializing in the application of digital computers to the control of processes. They are clearly based on considerations of radar systems, but are equally applicable to the control, by sampling, of any continuous system in which the value of the controlled variable between sampling periods is of as much interest as the sampled values.

The first of these, by V. P. Perov, concerns the determination of control parameters for system optimization, in a least-squares sense. Using the methods of the z-transform and the variational calculus, he first obtains general conditions for minimization of error at each sampling point, using variable parameters; these conditions, however, even under strongly simplifying assumptions, are practically unrealizable. He then finds conditions for minimization, with fixed parameters, of average error over a given time interval, and gives examples in simple cases.

The second paper, by F. M. Kilin, goes into methods of analysis of transient and steady-state processes in pulsed systems, in which the pulses are not necessarily rectangular, and of response to random signals. Each of these papers comprises a highly detailed, extensive and painstaking presentation of pulsed system theory, which almost makes a textbook in itself.

There are two papers on vibrational smoothing of non-linear characteristics, by self-imposed and by forced oscillations, two on special non-linear systems, one on solution of a particular type of non-linear differential equation, and one on optimum bang-bang control subject to power limitation. The difficulties caused by different national conventions in terminology are well illustrated by the last paper, which is entitled "Some questions relating to the logical design of circuits and the selection of characteristics for high-speed servo-mechanisms."

Finally, there is a most intriguing paper on the theory of escapement regulations, the 61 pages of which must constitute the most thorough theoretical investigation of the dynamics of clock mechanisms yet made, and which includes an excellent example of the use of phase-plane methods in complicated cases.

Throughout, translation is of excellent quality. Misprints exist, but not in such number as to be intolerable. The photo-reproduction of type-face is good in the wording, but causes eye-strain in some of the very complicated mathematical notation, involving subscripts and superscripts of tiny size and sometimes faint and ragged reproduction.

R. H. TIZARD.