

Iterative analogue computation applied to inventory policy simulation

By C. D. Lewis*

Although orthodox analogue computer techniques have been applied extensively in industry to Process Control, the analogue computer has made little impact in other fields of industrial computation such as Production Control. This paper discusses iterative analogue computing techniques, as opposed to hybrid, and the application of these techniques in performing simulations of some basic inventory control policies, such as the Re-order cycle, Re-order level and s, S policies, and suggests that in this form the analogue computer may have applications in a field where so far it has been little used.

Although control of continuous processes through servo-operated mechanisms has provided the analogue computer with one of its most fruitful markets in recent years, in other fields of industrial computation, such as Production Control, the analogue computer has made little impact, especially since the general acceptance of digital computing techniques. A few applications have been made in off-line computations of a simple arithmetic nature where solutions are required immediately on site and, with the introduction of methods of solving small linear-programming problems, solutions of the "Blending" problem have been carried out on a limited scale in the oil, powder and granular product industries (Lewis, 1963).

The basic reason for the lack of application of analogue computers in the field of Production Control would appear to be that the data and problems involved are not generally of a continuous nature, and hence not suitable for solution by the analogue computer in its traditional role as a differential analyzer.

The digital computer, however, as an iterative device has become an essential tool of many large Production Control departments; why then should not the analogue computer, used in an iterative mode, also have applications in this field with its advantages of low cost of equipment, rapid display of information and ready access?

The relative inaccuracy of the analogue computer, as compared with the digital, need not be a limiting factor, since, in Production Control, many of the problems dealt with have data whose value is known with less than 5% accuracy.

Basic principles of:

(a) Iterative computation

The iterative analogue computer differs fundamentally from the hybrid computer in so far that *all* functions such as storage and logic are based on analogue signals. Hybrid computers, however, consist of varying propor-

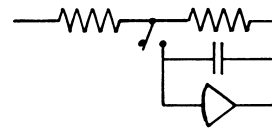


Fig. 1a.—Hold

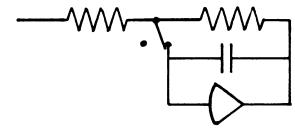


Fig. 1b.—Reset

tions of both digital and analogue equipment, with signal storage being performed almost invariably in a digital core store, necessitating analogue-digital and digital-analogue conversion (both for actual and logic signals).

The price of the hybrid computer, which for economic reasons would not usually have less than 200 operational amplifiers in the analogue portion, tends therefore, to be of the same order as a small digital installation. The iterative analogue computer, however, consisting of traditional analogue components with a small amount of control mechanism, costs little more than the conventional analogue computer.

The basic component of an iterative analogue computer is the integrating amplifier, used not as an integrator with respect to time but as a storage device. In this mode the amplifier is used only in the two configurations known normally as HOLD and RESET, as shown diagrammatically in Figs. 1a and 1b.

Using amplifiers in these two configurations, as controlled by relay chains (relays momentarily operating in sequence) one can perform both addition to and subtraction from a running total, as is shown in Fig. 2 using a two-relay chain.†

Referring to Fig. 2; with the momentary operation of relay 1, contacts 1/1 store $-1v$ in Amp. 1 which is then introduced as $+1v$ into Amp. 2 by contacts 2/1 when relay 2 operates momentarily. At the next operation of the relay chain, contacts 1/1 store the sum of the fixed $+1v$ and the output of Amp. 2 which is also $+1v$, and which with inversion, therefore, becomes $-2v$ in Amp. 1. Contacts 2/1 then store this value as $+2v$ in Amp. 2 in

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† All relays now shown unoperated.

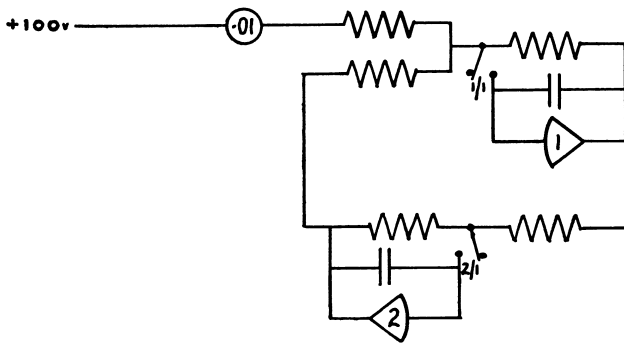


Fig. 2.—Addition using two-relay chain

readiness for the next computation cycle, and the addition of 1v to a running total is achieved at each operation of the relay chain. With a negative input, subtraction rather than addition occurs.

Hence, it can be seen that using a control relay chain and additional relays presenting voltages of either polarity at specified times, one can perform iterative operations on the analogue computer whilst still retaining normal analogue computer facilities such as continuous division, multiplication and integration with respect to time.

(b) Simulation of information for Stock Control problems

In developing a simulation model to study the effect of inventory control policies, one relies on reproducing past data (or information proportional to it) and using this data as input information to a model representing the actual situation existing. To validate the model one is required to obtain results that correlate well with actual past results, when model parameters are set at the same value as those known to exist in the real situation. Once the model has been proved in this way, model parameters can then be varied to study what conceivably would have happened (within certain statistical limits) in the real situation, had the actual parameters been varied similarly.

In a Re-order cycle policy (Newberry, 1960), demand orders for inventory of random size are placed on the supplier at random intervals of time. These demand orders are usually assumed to be fulfilled immediately and are thus deducted from the inventory held. At fixed intervals of time, the inventory level is reviewed and a replenishment order placed, which, were there no lead-time (delay) would bring the inventory held up to a fixed maximum level. However, because of the lead-time, replenishment orders are delivered sometime after being placed, and it is at this stage that the inventory may fall to zero, i.e. run-out occurs.

Should run-out occur, the supplier has a choice of either refusing further demand orders, or accepting orders and fulfilling these when the replenishment order arrives. The latter is known as *backordering*.

The data therefore required as input for the simulation model, and over which the supplier has little or no control, is the distribution of demand orders and lead-time durations. Both of these, being stochastic variables, would normally be defined by means of histograms.

Parameters, over which the supplier has full control, are the frequency at which the inventory level is reviewed, the value of the arbitrary maximum level chosen when computing the value of the replenishment order, and the choice of whether to backorder or not. These three then are the control parameters to be built into the simulation model.

Analogue program for inventory policy simulation

(a) Generation of stochastic input data

For ease of generation, it was assumed in this model that all demand orders occurred at regular intervals of time (but not necessarily at every interval) and that when received orders were fulfilled immediately. This was not considered too artificial a restriction as in practice orders often arrive simultaneously through the post or can be held for a period of time and then dealt with collectively. Having made this basic assumption it is then only necessary to generate one demand order per unit time.

The basic unit of time of the iterative analogue computer is the operation-cycle of the control relay chain, so to generate one demand order per cycle one requires only an additional sampling relay in the control relay chain to sample and store a voltage proportional to the demand order value. This brings the number of relays in the control chain up to three, and a separate sampling relay is used to derive lead-time duration values, as will be discussed later.

The value of the voltage stored by these sampling relays is derived from a modification of Wilkins' Random Step Generator (Wilkins, 1964). The principle of operation of this device is that if a Dekatron with ten separate cathodes is driven by a random voltage source such as a thyratron, even with regular sampling the probability that the glow will land on any particular cathode will, on average, be 10%. If potentiometers are used as cathode loads, two at the same setting will ensure that that particular voltage has, on average, a 20% probability of occurrence. Using two randomly driven Dekatrons and summing the two outputs, by the same method of reasoning it is possible to generate voltages with a 1% probability of occurrence.

Using a two-Dekatron version of this Step Function Generator (sampling electro-mechanically with a sampling relay rather than electronically as originally designed) histograms with 20 discrete levels have been generated with a goodness of fit better than at the 20% level of rejection as shown by the Kolmogorov-Smirnov one-sample test. Values are generated with a 1% accuracy which combines well with the general order of accuracy of analogue computers.

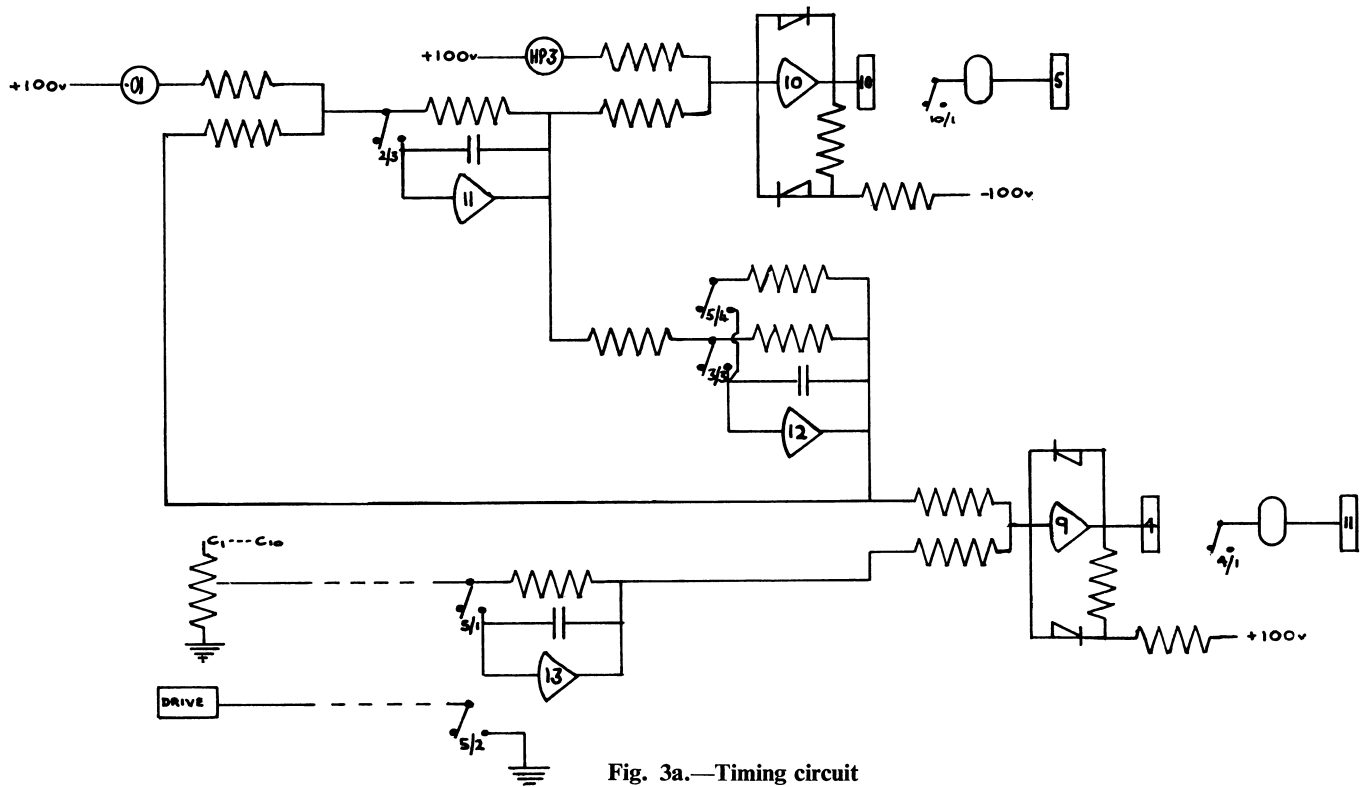


Fig. 3a.—Timing circuit

(b) Timing circuit* (Fig. 3a)

As has been shown earlier, by using a two-relay control chain (relays 2 and 3) and two integrating amplifiers (Amp. 11 and Amp. 12), addition of a fixed voltage to a running total can be effected. When this running total (output of Amp. 11), representing elapsed time, reaches the value of the re-order cycle as set by potentiometer HP3, Amp. 10 programmed as a discriminator operates relay 10 whose contacts 10/1, actuating a flip-flop circuit, momentarily operate relay 5. With this momentary operation Amp. 12 is reset to zero by contacts 5/4 and the cycle is repeated at a frequency controlled by potentiometer HP3.

Simultaneously with the resetting of time to zero, contacts 5/1 and 5/2 sample and store a random lead-time value derived from a random step generator (C_1, \dots, C_{10} , assuming a single Dekatron circuit) in Amp. 13. When elapsed time (output of Amp. 12) reaches this lead-time value, Amp. 9 programmed as a discriminator operates relay 4 whose contacts 4/1, through a flip-flop, momentarily operate relay 11.

Hence the timing circuit provides a momentary operation of relay 5 at the end of each re-order cycle, and a momentary operation of relay 11 at the end of each lead-time duration. A restriction imposed by this circuit is that the lead-time must always be less than the re-order cycle.

* For ease of description, it is assumed that whenever demand order, replenishment order, re-order cycle, lead-time or elapsed time are mentioned in the text a DC voltage proportional to these quantities is meant.

(c) Inventory circuit (Fig. 3b)

With the operation of the three-relay chain (consisting of relays, 1, 2 and 3) each demand order cycle, contacts 1/3 and 1/4 sample and store a negative demand order derived from a further random step generator (C_{11}, \dots, C_{20}) in Amp. 1. This demand order is then subtracted by contacts 2/2 and 3/2 from the inventory level in Amp. 2 and Amp. 3.

At the end of each re-order cycle, as indicated by the momentary operation of relay 5, contacts 5/3 store a replenishment order in Amp. 4, the value of which is equal to a maximum level as set by potentiometer HP9 less the inventory level at the time of placing (output of Amp. 3).

With the ending of the lead-time, as indicated by the momentary operation of relay 11, the replenishment order (output of Amp. 4) is presented positively to the input of Amp. 2 by contacts 11/2, to be added to the inventory level by contacts 2/2 at the next demand order cycle simultaneously with the subtraction of the fresh demand order.

Immediately after the addition of the replenishment order to the inventory level, Amp. 4 is reset to zero by contacts 11/1 and 3/1 in series. This is not an essential operation, as Amp. 4 is reset to the next replenishment order value at the end of the re-order cycle; but the resetting of Amp. 4 to zero at the end of the lead-time does provide a better indication of lead-time duration when monitoring.

The choice of backordering or not is simply achieved by constraining the polarity of the inventory level.

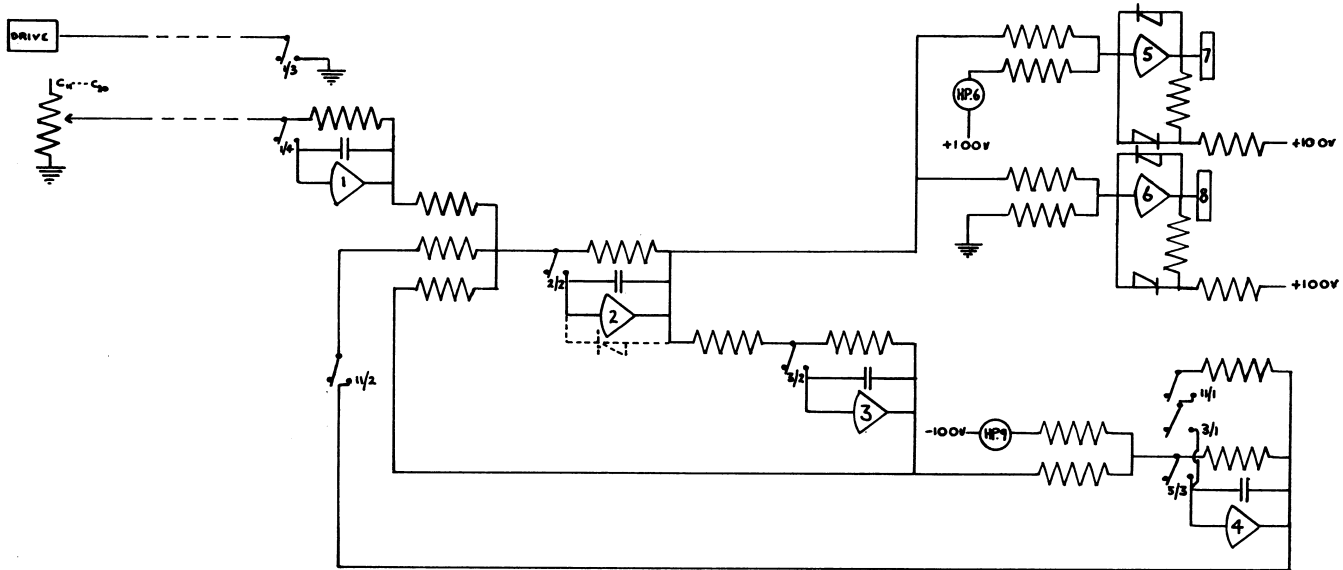


Fig. 3b.—Inventory circuit

Should backordering be required Amp. 2 and Amp. 3 are allowed to change polarity, and if backordering is not allowed, a diode in the feedback of Amp. 2 prevents either amplifier changing polarity.

The inventory circuit then performs the arithmetic manipulations defined by the re-order policy, allows the alteration of both the re-order cycle duration (HP3) and the arbitrary level from which the replenishment order is calculated (HP9), and allows the choice of whether to backorder or not to be made (diode feedback Amp. 2).

Results achieved by the analogue simulation

In measuring the effectiveness of any inventory policy, three of the main factors required are:

- the mean (average) inventory level,
- the risk of runout,
- the utilization of safety stocks.

In the analogue simulation the mean inventory level can be evaluated by means of a moving average. The moving average is computed by integrating with respect to time the inventory level and continuously dividing this by the integral of time as represented by a ramp function.

The frequency of runouts and utilization of safety stocks can both be measured by monitoring the inventory level with discriminators (Amp. 5 and Amp. 6) and counting the number of times zero and the safety level are broken. In addition, the time for which the discriminator relays operate gives a measure of the duration of runouts and safety stock utilization.

Results so far produced by analogue simulation have corresponded well with those calculated either digitally or manually.

The speed at which an iterative analogue computer can be run is dependent primarily on the charging time of the integrating amplifiers in the reset condition, which

using 100 Kohm resistors and 0.1 micro-farad capacitors should not be less than about 80 milliseconds. Using a three-relay chain one is therefore restricted to an iteration cycle of a third of a second when allowing time for independent operations to take place between iterations.

With only a few more amplifiers, it is relatively simple to extend the scope of the simulator to other inventory policies such as:

1. The Re-order level policy; where fixed value replenishment orders are placed when the inventory level falls below a previously stated level.

2. The s, S policy; which is similar to the Re-order cycle policy with the added criteria that replenishment orders are only placed if, at review, the inventory level is below a previously stated level s .

Photographs showing simultaneously demand orders, inventory level (with respect to a zero base line) and replenishment orders, with backordering prohibited, for these two policies and the Re-order cycle policy are given as Figs. 4a, b and c.

Using the explanatory diagram Fig. 5, points worth noting from these photographs are:

1. Correlation between zero demand and zero drop in inventory level.

2. Variable lead-times, $t_{ri} - t_{pi}$.

3. Occurrence of runouts.

4. For the Re-order cycle and s, S policies, regular review periods $t_{pi} - t_{pi-1}$ (or multiples thereof in the case of the s, S policy) but varying sized replenishment orders.

5. For the Re-order level policy, varying review periods, but fixed replenishment orders.

Conclusion

Iterative analogue computing is a method of using standard analogue computing elements to perform simple iterative operations. At present, speeds of

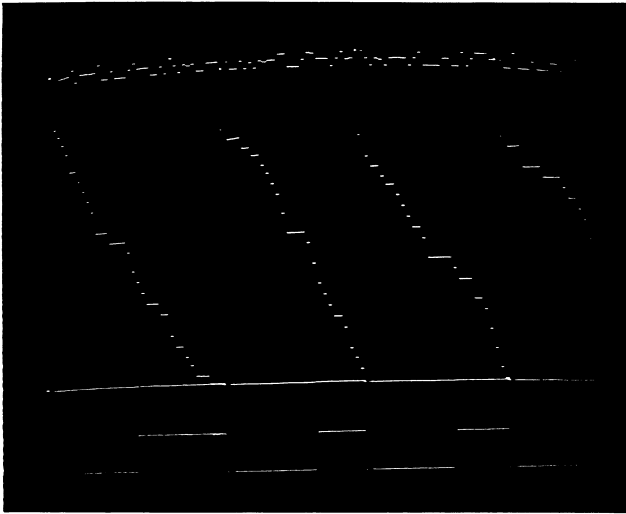


Fig. 4a.—Re-order level policy

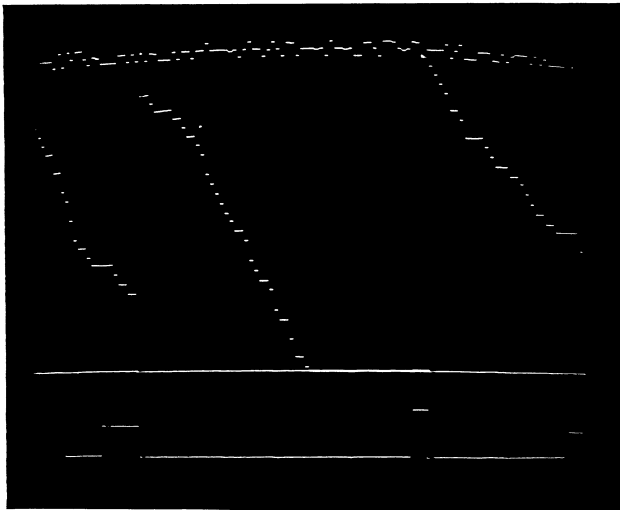


Fig. 4b.— s, S policy

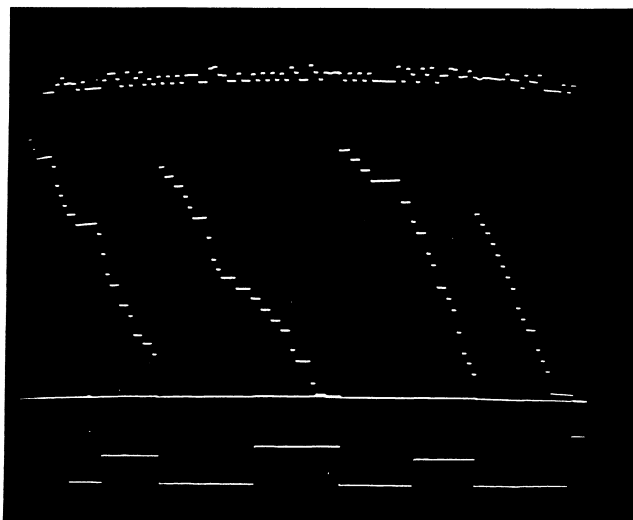


Fig. 4c.—Re-order cycle policy

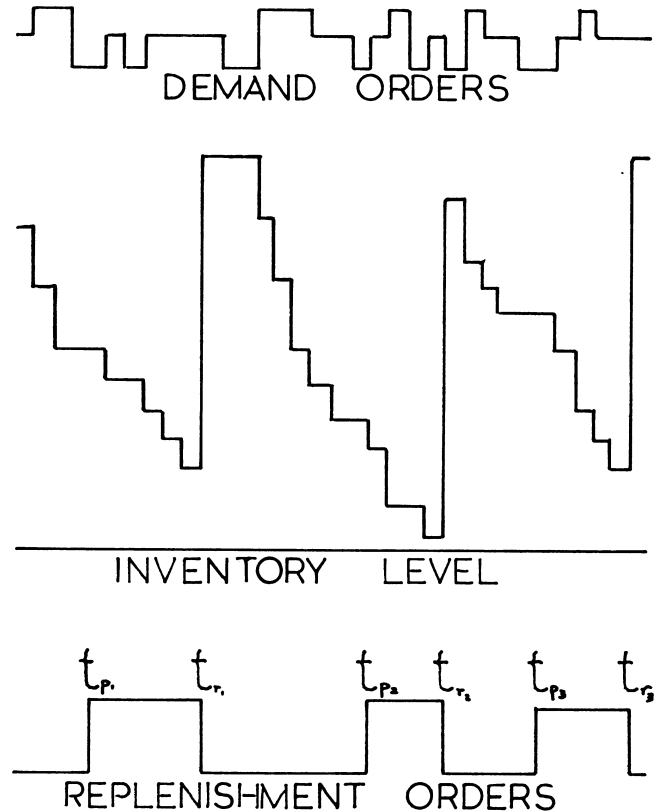


Fig. 5.—General inventory situation

iteration are slow, but already solid-state DC amplifiers are being developed capable of 1000 iterations per second (Korn, 1964) so that in a few years time this restriction need no longer be limiting.

Many of the problems that do occur in Production Control are multi-product problems, and the consequent replication of equipment required for iterative analogue solution may often not be justified. There are, however, many problems involving few products, arithmetically simple but with complex stochastic data, whose solution with very high accuracy by digital means cannot be justified, especially when that data is not defined with any great accuracy. It is for this type of problem that the iterative analogue computer solution could provide a speedy and reasonably accurate answer.

This paper has dealt exclusively with the use of an iterative analogue simulation of an inventory situation, but with different control circuitry many problems both within and outside the field of Production Control can be tackled. It is hoped that further research will provide more flexible control logic and hence widen the application of this technique.

Acknowledgement

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Book Review

Pulse Technology, by WILLIAM A. STANTON, 1964; 255 pages. (London and New York: John Wiley and Sons Ltd., 53s.).

Once the author's preface is read and the objective of the book is fully understood, the initial impression of disappointment formed after a cursory examination of the text is rapidly dispelled. This would suggest that a more explanatory title such as "The Fundamentals of Pulse Technology" would be more appropriate than the broad title the book now possesses.

Although the book is primarily intended for the laboratory technician, the author hopes that it will prove of value to those engineers and scientists who need to know the fundamentals of electronic circuits. One feature of the work is that the level of the mathematics involved is such that a knowledge of calculus is not essential. Within these constraints, the author has produced a readable, lucid and concise introductory textbook on the basic essentials of pulse technology.

The first chapter is devoted to a review of fundamental circuit theory. The main differences between the treatment of sine and pulse waveforms are clearly explained. Thevenin's and Norton's equivalent network theorems are adequately covered, aided by examples and applications of these theorems to circuit analysis. Circuit parameters of transistors are discussed leading to comparisons of the operation of these devices and of thermionic valves as amplifiers. The effect of negative feedback in amplifiers and the advantages gained by its use are also clearly explained. The chapter closes with a short and useful commentary on that most essential tool of the circuit engineer, the oscilloscope.

Wave-shaping techniques are outlined in Chapter II, covering the classical methods of clipping, differentiation, integration and clamping.

Multivibrators are covered fairly exhaustively in Chapter III. The operation cycles of both valve and transistor versions of free-running, monostable and bistable multivibrators are explained by means of waveform diagrams. A similar technique is used in Chapter IV dealing with pulse generation, in which the operation cycles of (a) blocking, (b) shock-excited and (c) phase-shift oscillators are discussed. The chapter concludes with sections on sweep generators and the Schmidt Trigger.

The use of the circuit elements so far studied is treated in Chapter V for switching and counting applications. The operation of chains of bistables with interstage feedback is lucidly explained for a number of typical counting and timing scales. Methods of display are also treated, including a description of the operation of the cold-cathode Dekatron.

No introduction to pulse technology would be complete without some reference to number systems and coding and the elements of logic, so the author has covered these subjects in the next two chapters. In Chapter VI conventional methods of conversion from one number system to another are given with typical uses of systems of different radix. A section on arithmetic operation in binary completes the chapter. Chapter VII contains a concise but clear exposition of the basic rules of Boolean algebra, developed with the aid of Venn diagrams. The use of this algebra is then considered in terms of circuits for performing various logical functions, illustrated by examples in code conversion.

A brief introduction to computer programming is given in the final chapter again clearly illustrated by two examples of simple programming. The book is rounded off with a glossary, a useful bibliography on pulse technology and three appendices. The first two of the latter are added no doubt as a sop to the more mathematically-minded reader. The first covers the use of complex quantities for solving AC network problems, while the second is a thumbnail outline of calculus and its application to circuit theory. The third appendix is an extract from the U.S. National Bureau of Standards' List of prefixes and symbols.

The method of presentation of the book is called by the author the lesson plan approach, in which, once essential facts have been presented and discussed in some detail, repetition is achieved by consideration of examples. Furthermore, each chapter is followed by a series of questions, quizzes and problems, designed to high-light the main points of the chapter. Some of these are of the restricted answer type; others to determine the nature, false or true, of given statements. Most of the problems are vehicles for the introduction of formulae and degenerate into arithmetical exercise. Unfortunately, there are a few misprints and errors. Problem 5 of Chapter I introduces the formula for the deflection of an electrostatic cathode-ray tube. The parameter d_s is given as the "deflection sensitivity," whereas it should be the "deflection." Some also occur in the answers at the end of the book, the magnitude of which are such that they cannot be attributed to rounding-off or slide-rule inaccuracies. These would be unimportant when the book is used as a text in a class room but would be disturbing to a student working on his own.

However, apart from these minor criticisms the author has produced an admirable primer on pulse technology, which more than adequately attains the objectives outlined in the Preface.

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