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Book Review

Integration of Equations of Parabolic Type by the Method of Nets, by V. K. Saul'yev, 1964; 346 pages. (Oxford: *Pergamon Press Ltd.*, 80s.)

This useful book is essentially a practical guide to the numerical solution of parabolic equations, and incidentally of elliptic equations also, by finite-difference methods. The first part of the book, accounting for rather more than half its length, is devoted to the finite-difference approximation of parabolic equations; that is, to the replacement of a parabolic equation by a system of algebraic equations. Systems of explicit and mixed types are derived, and while the emphasis is naturally on uniform nets, the non-uniform net is also considered, as is the net with fictitious nodes (to deal with irregular boundaries). Most of the formulae concern problems with one or two space variables, but the three-dimensional case is discussed; indeed most situations of common practical occurrence are covered. There is a section on equations of order greater than two, and one on nonlinear equations. Many contributions of the author and his associates appear in this part of the book; some will probably be new to Western readers. The stability of all formulae is considered, and the truncation error is examined at all relevant stages.

Part II is concerned with the solution of the systems of algebraic equations derived in Part I. Since explicit formulae are solved trivially, the methods discussed are those for solving an implicit system; in the important two-dimensional case they are essentially methods for solving, at each interval of time, the system arising from an *elliptic* equation. Direct methods are dismissed early in the discussion on the ground that they usually demand too much arithmetic. A comparison of various iterative methods therefore dominates this part of the book. There are accounts of the Jacobi and Gauss–Seidel methods, successive over-relaxation, variational methods (including the method of steepest descent and the method of conjugate gradients), methods using Chebyshev polynomials, and methods using block iteration (including alternating-direction methods). There are also sections on iterative methods of second and higher degrees (a method of nth degree being one in which the evaluation of the mth approximation $u^{(m)}$ to the solution requires knowledge of $u^{(m-1)}, u^{(m-2)}, \ldots, u^{(m-n)}$.) Questions of error, convergence, amount of computation and computer storage space required are discussed fully.

There are many references, particularly in Part I, to papers dealing more fully with the theoretical background to some of the methods. Many of these references are, as might be expected, to Russian sources, but several of the more important Western works have been added to this translation.

Part I covers its subject-matter admirably, and if Part II is not quite up to date, this is perhaps understandable in a survey of a subject which is still developing rapidly. The reader might consult R. S. Varga, *Matrix Iterative Analysis* (Prentice-Hall, 1962) for a more complete account. The translation is generally very good; seldom does the style remind the reader that it *is* a translation. The title of the book might be misleading, in that "method of nets" will be a term unfamiliar to many people, and there may be a suspicion that a basically *new* method is being proposed. The author's preference to restrict the term "method of finite differences" to the corresponding method for *ordinary* differential equations is difficult to understand, and hardly likely to find general acceptance.

The author has not set out to make his book completely comprehensive. He mentions topics which are *not* covered; they include problems with general boundary conditions and problems with moving boundaries. He says "The present book is designed for a wide class of readers, having direct or indirect contact with the numerical solution of parabolic and elliptic net equations (particularly heat conduction and Laplace's equation)." The reader who is often confronted with such problems will be very grateful for this volume, which provides the desired information with all the necessary warnings.

C. W. Clenshaw