*To the Editor,*
*The Computer Journal.*

Sir,

Mr. ApSimon has misunderstood the nature of *reductio ad absurdum* proofs; these depend on the theorem that if a hypothesis leads by a valid argument to a contradiction it must be false. It is important, of course, that the argument need only be valid (and indeed could only be valid) if the hypothesis were true; as the whole point of the proof is to show the hypothesis to be false, the argument is never in fact applicable—it merely would be if the hypothesis were true.

The second and third of Mr. ApSimon's refutations consist of pointing out places where the chain of argument leading to a contradiction would break down if the hypothesis were false; if such places did not exist we should be in the unfortunate position of having produced a paradox instead of a proof, and others besides Mr. ApSimon might feel disturbed.

At first sight, it looks as if Mr. ApSimon's first refutation arises from careless reading. Although I *define* the fraction $T[R]$ I preface the definition with the word "suppose" so that it is clearly a hypothetical fraction, and to say that it *exists* by definition is merely playing with words. However, there is a real philosophical problem about the possibility of defining non-existent objects though I know of no mathematician who would deny the validity of proofs involving such a step. If Mr. ApSimon wishes to adopt this very radical position, he should come clean and say so—and at the same time, I should be grateful if he would tell me if he accepts the validity of any non-existent proof; for example, how would he prove that $\sqrt{2}$ is an irrational number?

Yours faithfully,

C. STRACHEY.

Churchill College,
Cambridge.
May 1965.

## Integration of differential equations

*To the Editor,*
*The Computer Journal.*

Dear Sir,

In connection with the excellent method suggested by J. T. Day for the numerical integration of differential equations of the type

$$y'' = f(x)y + g(x),$$

which appeared in this *Journal* (Vol. 7, pp. 314–317), we would like to point out an unfortunate mistake. In the definition of $\Delta$, the "1" placed in the numerator should be a free term, so that the correct formula is

$$\Delta = 1 + \frac{h^2p^2f(x_p)(p-3q)}{6(q-p)} + \frac{h^2q^2f(x_q)(3p-q)}{6(q-p)} + h^4f(x_p)f(x_q)/432. \quad (2.7)$$

The constant "432" is obtained using the numerical values of $p$ and $q$ and some of the coefficients are derived using the relation $q = 1 - p$ (otherwise the formulae would also hold for other values of $p$, $q$).

We would like to point out that by substituting the actual values of $p$ and $q$ throughout, the formulae become

$$u_p = y_n\left[1 + \frac{h^2}{36}(4-3\sqrt{3})f_q\right] + hy_n'\frac{\sqrt{3}-1}{6}\left[\sqrt{3} - \frac{h^2}{12}f_q\right] + \frac{h^2}{36}\left[g_p + (5-3\sqrt{3})g_q - \frac{h^2}{12}f_qg_p\right],$$

$$u_q = y_n\left[1 + \frac{h^2}{36}(4+3\sqrt{3})f_p\right] + hy_n'\frac{\sqrt{3}+1}{6}\left[\sqrt{3} + \frac{h^2}{12}f_p\right] + \frac{h^2}{36}\left[g_q + (5+3\sqrt{3})g_p - \frac{h^2}{12}f_pg_q\right].$$

However, this stage may be omitted altogether. The final formulae for $y'_{n+1}$ and $y_{n+1}$ are then

$$y'_{n+1} = y'_n + \frac{h}{2\Delta}\left\{y_n\left[f_p + f_q + \frac{2h^2}{9}f_pf_q\right] + hy'_n\left[pf_p + qf_q + \frac{h^2}{36}f_pf_q\right] + A_1\right\},$$

$$y_{n+1} = y_n + hy'_n + \frac{h^2}{2\Delta}\left\{y_n\left[qf_p + pf_q + \frac{h^2}{36}f_pf_q\right] + \frac{hy'_n}{6}[f_p + f_q] + A_2\right\},$$

where $f_p = f(x_p)$ etc. and

$$\Delta = 1 - \frac{h^2}{36}\left[f_p + f_q - \frac{h^2}{12}f_pf_q\right],$$

$$A_1 = [g_p + g_q] + \frac{h^2}{36}[(4+3\sqrt{3})f_qg_p + (4-3\sqrt{3})f_pg_q],$$

$$A_2 = [qg_p + pg_q] + \frac{h^2}{72}[(1+5\sqrt{3})f_qg_p + (1-5\sqrt{3})f_pg_q].$$

If $g(x) = 0$ (as in the examples shown in the paper) $A_1$ and $A_2$ are zero.

It appears that these equations have decided programming advantages. For small machines only $\sqrt{3}$ has to be stored, whereas on larger machines the integration can be performed very fast indeed by storing a few precalculated coefficients. We tried the method using these equations and obtained similar results to those published. Indeed, these formulae appear to give slightly reduced rounding errors.

We would like to emphasize that the method is very attractive. Although it does not refer to any previously integrated points it seems to be more accurate than other available methods.

Yours faithfully,

G. J. COOPER, E. GAL.

Institute of Computer Science,
44 Gordon Square,
London, W.C.1.
23 June 1965.