

The underground storage of compressed air for gas turbines: a dynamic study on an analogue computer

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The usefulness of a gas-turbine driven alternator for generation of electricity would be enhanced if air could be compressed during the off-peak period for use during the peak period. This analogue study was made to examine the physical behaviour of an underground tunnel used to store the compressed air. The physical system and the analogue are discussed to make it clear what assumptions have been made. Graphs are included to illustrate the results, the most important of which relate to leakage of air through the rock, and the maximum air temperature attained in the tunnel. The air leakage is important because it affects the efficiency of the system, and the maximum air temperature decides the size of tunnel required to hold enough air for a given size of station.

1. Introduction

The usefulness of gas turbine generators during peak loads would be enhanced if part of the power were not used to compress air for the gas turbine. This could be achieved if the alternator were used as a motor during off-peak periods to drive the turbine compressors, and provided that a means could be found of storing the compressed air. It has been proposed that a tunnel of suitable volume be bored in rock for this purpose. This tunnel could be pumped up by a smaller compressor than would be necessary to supply the turbine because the compressor could take longer to perform its task, the limit being set by the duration of the cheap, off-load, electricity period.

The analogue study is of the behaviour of the tunnel during the working of such a scheme.

2. The scheme for the study

One of the difficulties of the model to be studied is that to make the model useful for study of certain parameters, assumptions have to be made about the behaviour of others, which may restrict their behaviour. These assumptions are pointed out below as they are described.

The tunnel was to be pumped up to 50 atmospheres each day and during the generation period was to be decompressed to 18 atmospheres. The air flow for each of these processes was assumed to be at constant mass/unit time. This is not necessary in practice but would probably be so, and some assumption had to be made for the model.

The coolers on the compressor are placed at various stages before the constant mass flow throttle, and the temperature regulating element placed between the coolers and the throttle. Providing the kinetic energy of the air leaving the throttle is destroyed in the tunnel the temperature of the air before and after the throttle are the same (assuming a perfect gas law obtains, see

Section 3). Thus a further assumption could be made that the air could be supplied to the cavern at constant temperature. This is not necessarily so in practice; indeed there may be reasons for deliberately varying it, but it provides a basis for the model.

An attempt was made to overcome this complication by making some studies in which only the temperature of air entering the cavern was altered. (The highest temperature of 650°C represents air that has not been cooled at all after leaving the last stage of compression.)

The tunnel was to supply air for a 200 megawatt station. The actual arrangement of tunnel volume, i.e. one long tunnel or a number of branches, was not decided at the commencement of this study, because it is dependent on the geology of the site chosen. One possible arrangement suggested has four parallel tunnels, twenty-five feet in diameter lying at a separation of about 3 tunnel diameters. These lie roughly 700 feet below ground; the ground being a hill top.

3. The physical problem

One of the difficulties of finding a physical model of the tunnel was the uncertainty about how the air would circulate and mix.

Complete mixing would be quite easy to describe but difficult to justify in a tunnel two miles long and 25 feet in diameter, or half a mile long in the case of a four-tunnel scheme. On the other hand a model where the air did not mix but moved along the tunnel only as more air entered to compress it appeared even more unrealistic. It seemed likely that an intermediate state, rather difficult to describe, would obtain. Since this state would have intermediate maximum air temperatures and other properties, there was some justification for examining both models.

It has been pointed out that if, in the four-tunnel system, a connecting tunnel were bored across the ends, the kinetic energy of the inflowing air may produce a

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circulation sufficient to mix the air. Some calculations show that the air would circulate about ten times during a nine-hour compression period. This would probably be sufficient for the system to behave like the mixing model. For the purpose of this study therefore complete mixing of the air was assumed.

One of the advantages conferred by the mixing model is that, since all parts of the tunnel system are the same, behaviour along the length of the tunnel need not be described. (End effects are ignored anyway because of the proportions of the tunnel.) This frees analogue equipment for an improved simulation of some other part of the model.

In the study that was made, a radially symmetrical model of heat and air exchange between the rock surface and the tunnel was used. This has the advantage that the equations describing heat and air movement have only one space dimension. The equations also have a simplifying symmetry.

For heat conduction in the rock this model is satisfactory since the temperature variations do not penetrate more than a few feet. The air movement in the rock, however, may well be significant at distances comparable with the distance to ground level, and so the symmetry becomes rather unrealistic. An analytic solution may be obtained for steady flow of air from the tunnel to the surface (i.e. with no pressure variations with time in the tunnel). Comparing this with the equation for steady flow to a cylindrical outer surface, we find that the air flow is about 17% greater for the cylindrical model with a 25 feet diameter cave at 700 feet below ground. However, some preliminary analogue studies show that it takes many weeks to achieve the steady state, so that in a problem where the interest is in diurnal variations the behaviour far from the tunnel becomes very much less important. Another way of stating the same fact is that the attenuation of pressure disturbances is a function of frequency. Thus very long-period disturbances are still perceptible at a distance comparable with the tunnel-to-surface distance, but diurnal variations do not penetrate very far.

The propagation of both temperature and air pressure disturbances in the rock depends on rock properties, and data on these effects are given in the section on the analogue results.

Two implicit assumptions arising from a radially symmetrical physical model are that the rock is isotropic and that the tunnel surface is smooth. The first of these is likely to be untrue because of stratification and fissuring in the rock: this will affect air leakage. The second is also untrue: the first finite-difference points for the heat-flow equation are 3 inches apart, and so surface irregularities of this order of size will affect the description of heat flow. This is not likely to be serious, however.

A further assumption which might be mentioned at this point is that the rock is dry. It is likely that the tunnel will be below the water table and so air flowing through the rock will have to displace water. Under

these circumstances it is the flow of water that regulates air movement because of its greater viscosity. Even if the air "blows" a conical chimney through the water in the rock, the air leakage would be considerably less than that predicted by the analogue. The pressure head of water, if the rock were saturated, would not exceed 25 atmospheres.

The rock surface temperature is not the same as the mean tunnel temperature, owing to air temperature variation in the tunnel. Superimposed on any movement of air along the tunnel is a convective circulation due to the air being cooled at the tunnel wall.

If the air has a simple two-cell circulation, flowing upwards in the centre of the cave and downwards at the walls, the radial symmetry of rock temperature distribution is upset. This was ignored and is a minor error of the model.

It is difficult to express this circulation analytically, but empirical expressions exist for some convective systems that give heat transfer as a function of air conductivity, viscosity, etc. These have much the same form and since at the commencement of the study there was not one available for convection in a hollow horizontal cylinder the expression for convection outside a horizontal rod was used. It is dimensionally correct and with a suitable choice of constant of proportionality it fitted the experimental data obtained from trials made in some disused tunnels.

It has been assumed that air leaking from the tunnel plays no part in heat transfer from the tunnel to the rock, and so has no effect on rock temperature. It is further assumed that the leaking air attains rock temperature as soon as it leaves the tunnel. This is to simplify the air pressure and air flow equations. Since most of the conducted heat travels no further than a few feet from the tunnel, and the first finite-difference point for air flow is at 10 feet, this is probably a reasonable assumption.

Associated with the problem of finding a physical model for the scheme are the difficulties of obtaining adequate data for the physical properties of the system. The worst of these are the data for permeability and porosity of the rock. There are two main reasons for this. The first is that the geological structure of a rock bed depends very much on the site, so that special local information is required, and the second that because air flow in the rock occurs over such large distances, small site investigations are very much influenced by end effects. Ranges of values of permeability and porosity were therefore chosen to include any that might be encountered in practice.

Because of these uncertainties there seemed little point in allowing for variation from unity of the compressibility factor of the air, and for the dependence on pressure of viscosity, thermal conductivity and specific heat of the air.

The specific heat of the air was assumed to be constant, i.e. independent of temperature. The variations that have been ignored in these are of the order of 2-3% over the range of pressure and temperature expected.

In fact, they largely cancel out in the function expressing heat transfer from the air to the rock.

The best available data were used for assessing the thermal properties of the various possible types of rock. Slight variations occurring for a practical case could be obtained by interpolation.

Another difficulty was that of finding a suitable constant coefficient for the heat transfer function. The best figure available was obtained from the field trials, but the wetness of the cave and the difference in the ratio of length to radius compared with the proposed tunnel make the value unreliable. Because of this another smaller value was also used.

4. Equations describing the physical model

4.1 Heat transfer from air in the tunnel to the rock

The equation used for this was

$$\frac{dQ}{dt} = Bk_a(T - \theta_0)^{4/3}T^{-1/3}(\rho_a/\mu_a)^{2/3}2\pi r_0 l. \quad (1)$$

4.2 Cavern thermodynamics

The kinetic energy of the circulating air is negligible, and the drop in temperature at the throttle exit due to increase in kinetic energy is local and will disappear as soon as the air has moved away from the delivery pipe. The potential energy changes are also negligible. Leaving both these forms of energy out of the equation for heat transfer in an open system, the equation becomes

$$dH = \delta W + dU + \delta Q.$$

Since the external work done by the tunnel system is zero the equation may be written

$$\delta Q = dH - dU$$

where $dH = hdm$.

Consider this equation now in various phases of operation of the storage scheme, and assume the properties of a perfect gas.

Properties of tunnel air are unsuffixed.

(a) Leakage alone (suffix₃ indicates leakage)

$$hdm = (u + RT)dm_3$$

$$dU = d(um) = udm_3 + mdu$$

$$\begin{aligned} \therefore \delta Q &= RTdm_3 - mdu \\ &= RTdm_3 - m \frac{du}{dT} dT \end{aligned}$$

$$\frac{dQ}{dt} = RT \frac{dm_3}{dt} - mC_v \frac{dT}{dt}$$

$$\text{put } \frac{dm_3}{dt} = -F_3$$

$$\text{then } \frac{dQ}{dt} = -RTF_3 - mC_v \frac{dT}{dt}. \quad (2)$$

(b) Compression and leakage (suffix₁ indicates compression)

$$hdm = (u_1 + RT_1)dm_1 + (u + RT)dm_3$$

$$dU = d(um) = udm_1 + udm_3 + mdu$$

$$\begin{aligned} \therefore \delta Q &= (u_1 - u)dm_1 + RT_1dm_1 + RTdm_3 - mdu \\ &= C_p(T_1 - T)dm_1 + RT_1dm_1 + RTdm_3 - mdu \\ &= (C_pT_1 - C_vT)dm_1 + RTdm_3 - m \frac{du}{dT} dT \end{aligned}$$

$$\frac{dQ}{dt} = (C_pT_1 - C_vT) \frac{dm_1}{dt} + RT \frac{dm_3}{dt} - mC_v \frac{dT}{dt}$$

$$\text{put } \frac{dm_1}{dt} = F_1$$

$$\text{then } \frac{dQ}{dt} = (C_pT_1 - C_vT)F_1 - RTF_3 - mC_v \frac{dT}{dt} \quad (3)$$

(c) Decompression and leakage (suffix₂ indicates decompression)

$$hdm = (u + RT)dm_2 + (u + RT)dm_3$$

$$dU = d(um) = udm_2 + udm_3 + mdu$$

$$\therefore \delta Q = RTdm_2 + RTdm_3 - mdu$$

$$\frac{dQ}{dt} = RT \frac{dm_2}{dt} + RT \frac{dm_3}{dt} - mC_v \frac{dT}{dt}$$

$$\text{put } \frac{dm_2}{dt} = -F_2$$

$$\text{then } \frac{dQ}{dt} = -RTF_2 - RTF_3 - mC_v \frac{dT}{dt}. \quad (4)$$

Thus the whole of the thermodynamic processes in the tunnel may be represented by one equation provided F_1 and F_2 are put equal to zero at appropriate times.

The equation is

$$\begin{aligned} \frac{dQ}{dt} &= (C_pT_1 - C_vT)F_1 - RTF_2 \\ &\quad - RTF_3 - mC_v \frac{dT}{dt}. \quad (5) \end{aligned}$$

4.3 Thermal conduction in the rock

The equation for thermal behaviour of the rock is

$$\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} \frac{k_r}{\rho_r c_r}. \quad (6)$$

Since machine time is being reserved for representing time behaviour of the system, this equation must be represented by a system of finite-difference equations. Using central differences, the equation becomes

$$\begin{aligned} \frac{d\theta_n}{dt} &= \frac{k_r}{\rho_r c_r} \left\{ \left(\frac{\theta_{n-1} - 2\theta_n + \theta_{n+1}}{d^2} \right) \right. \\ &\quad \left. + \frac{1}{r_0 + nd} \left(\frac{\theta_{n+1} - \theta_{n-1}}{2d} \right) \right\}. \quad (7) \end{aligned}$$

The differential equation itself is of the Bessel type and it could be solved if $\theta(r)$ were known. It helps to know this because it means that the attenuation of the temperature wave in the rock, caused by a sinusoidal disturbance of temperature at the rock face, is a function of the number of wavelengths and not of absolute distance.

Thus for a periodic temperature disturbance of the sort produced by this system, the high-frequency components will not penetrate far from the rock face but the low-frequency ones may travel several feet before becoming negligible. This depends also, of course, on the thermal properties of the rock.

To provide the best description of the temperature behaviour of the rock, a system of finite-difference points was arranged at increasing intervals, because long wavelengths do not require the points so close together. The positions of the points were as indicated below.

θ_0	θ_1	θ_2	θ_3	θ_4	θ_5	θ_7	θ_9	θ_{11}	θ_{16}	θ_{19}	θ_{23}	θ_{31}	θ_{39}	θ_{47}
0	0.25'	0.5'	0.75'	1'	1.25'	1.75'	2.25'	2.75'	3.75'	4.75'	5.75'	7.75'	9.75'	11.75'

θ_{47} is fixed at the mean rock temperature; assumed to be 285° K.

For the remaining fourteen variables there are thirteen equations.

A further equation is obtained by considering heat conduction at r_0 :

$$\frac{\partial Q}{\partial t} = 2\pi r_0 k_r \left(\frac{\partial \theta}{\partial r} \right)_0.$$

Writing this in finite-difference form and introducing θ_{-1} , a fictitious finite-difference point on the tunnel side of the rock face, we obtain

$$\frac{dQ}{dt} = 2\pi r_0 k_r \left(\frac{\theta_{-1} - \theta_0}{2d} \right).$$

The central finite-difference form of the thermal diffusion equation for θ_0 may be written using θ_{-1} . It is

$$\left(\frac{\theta_{-1} - 2\theta_0 + \theta_1}{d^2} \right) + \frac{1}{r_0} \left(\frac{\theta_{-1} - \theta_1}{2d} \right) = \frac{\rho_r c_r}{k_r} \frac{d\theta_0}{dt}.$$

Eliminating θ_{-1} between these two equations we obtain

$$\frac{d\theta_0}{dt} = \frac{-2k_r}{\rho_r c_r} \frac{\theta_0}{d^2} + \frac{2k_r}{\rho_r c_r} \frac{\theta_1}{d^2} + \frac{1}{\rho_r c_r r_0 \pi l} \left(\frac{1}{2r_0} - \frac{1}{d} \right) \frac{dQ}{dt}. \quad (8)$$

$\frac{dQ}{dt}$ is defined by the heat transfer equation, T is defined by the thermodynamic equation, and tunnel air pressure is defined by the gas equation.

$$PV = mRT.$$

4.4 Air leakage from tunnel

The only term remaining undefined in this set of equations is the air leakage.

The diffusion equation for air in rock is

$$\nabla^2 P^2 = \frac{2f\mu}{k} \frac{\partial P}{\partial t}. \quad (9)$$

N.B. for k in darcys, P must be in atmospheres, length in centimetres and time in seconds.

This equation is even more difficult to solve than the heat equation, and again recourse is made to finite-difference methods. For a radially symmetrical system the equation becomes

$$\frac{\partial P}{\partial t} = \frac{k}{\mu f} \left\{ \left(\frac{\partial P}{\partial r} \right)^2 + P \frac{\partial^2 P}{\partial r^2} + \frac{1}{r} P \frac{\partial P}{\partial r} \right\}$$

which in finite-difference form* is:—

$$\frac{dP_n}{dt} = \frac{ck}{\mu f} \left\{ \left(\frac{P_{n+1} - P_{n-1}}{2\delta} \right)^2 + P_n \left(\frac{P_{n+1} - 2P_n + P_{n-1}}{\delta^2} \right) + \frac{1}{r_0 + n\delta} P_n \left(\frac{P_{n+1} - P_{n-1}}{2\delta} \right) \right\}. \quad (10)$$

For similar reasons to those applying in the case of the equation for rock temperature, finite-difference intervals are increased outwards, the final being at 1360 feet from the rock face—approximately twice the distance of ground level from the tunnel. The positions of the points are indicated below.

P_0	$P_{\frac{1}{2}}$	P_1	P_2	P_3	P_4	P_8	P_{12}	P_{20}	P_{28}	P_{36}	P_{52}	P_{68}
0	10'	20'	40'	60'	80'	160'	240'	400'	560'	720'	1040'	1360'

The pressure at P_{68} is fixed at one atmosphere, P_0 is fixed by the temperature and pressure of the tunnel air, and the remaining 11 point pressures are fixed by the finite-difference equation.

The equation for air flow into the rock is

$$\frac{dm_3}{dt} = 2\pi r l' \frac{k}{\mu} \left(\frac{\rho}{P} \right) \left(\frac{\partial P}{\partial r} \right)_0.$$

To obtain the expression for air leakage from the tunnel it is, unfortunately, not possible to use fictitious pressure at P_{-1} to obtain equations symmetrical about P_0 . This is because of the P^2 term in $\nabla^2 P^2$ giving such a complicated expression.

The unsymmetrical finite-difference equation obtained from the leakage equation is

$$\frac{dm}{dt} = c' 2\pi r l' \frac{k}{\mu} \left(\frac{\rho}{P} \right) \left(\frac{P_{\frac{1}{2}} - P_0}{\delta} \right) = F_3.$$

This completes the set of equations defining the system.

* This is not the only possible finite-difference system available; e.g. P^2 could be generated at each point so that

$$\nabla^2 P^2 = \frac{P_{n-1}^2 - 2P_n^2 + P_{n+1}^2}{\delta^2} + \frac{1}{r_0 + n\delta} \frac{P_{n+1}^2 - P_{n-1}^2}{2\delta}.$$

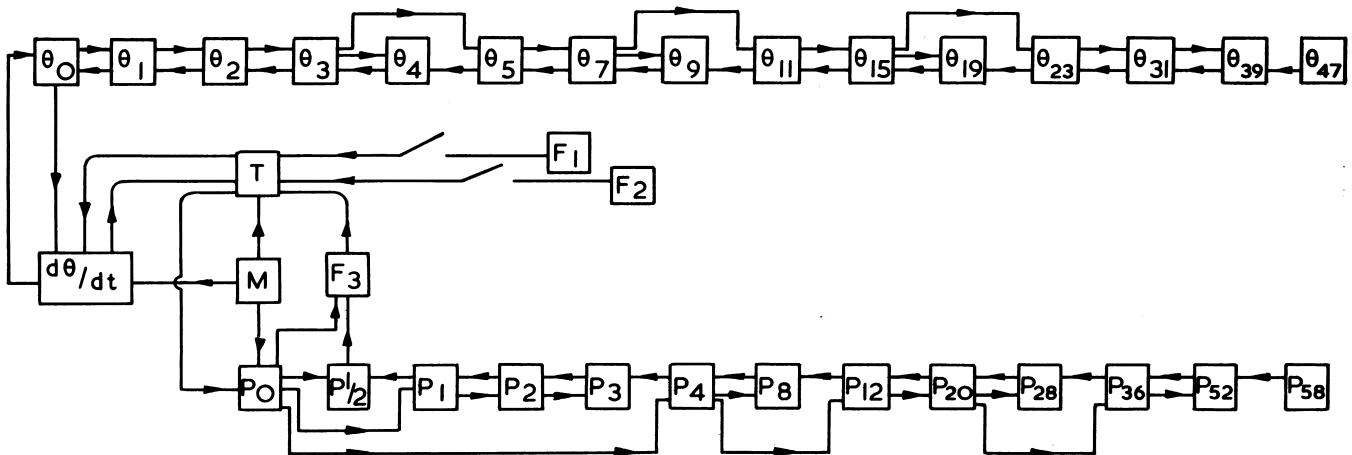


Fig. 1.—Block diagram of the analogue

5. The analogue study

5.1 The analogue

The finite-difference systems are represented in a straightforward manner by analogue equipment. **Fig. 1** shows a block diagram of the layout. Where there is a change of finite-difference interval, points are used to keep the finite-difference equations symmetrical, e.g. the points used with θ_5 are θ_3 and θ_7 . This seems to cause no noticeable lack of smoothness of temperature or pressure profiles in the rock. By way of illustration **Fig. 2** shows a typical unit of the pressure finite-difference system. Sign changers and attenuators have been omitted for the sake of clarity.

Generation of the special functions of equation (1) is tedious but not difficult. The switching arrangement used in this analogue is perhaps worth mentioning. It is shown in **Fig. 3**.

The part surrounded by pecked line produces a saw-tooth output from the integrator. The comparator switches operate both during the 23 second ramp and the one second return, but since the supply to F_1 and F_2 comes from the ramp generating voltage, there is no spurious compression or decompression when these switches operate the second time.

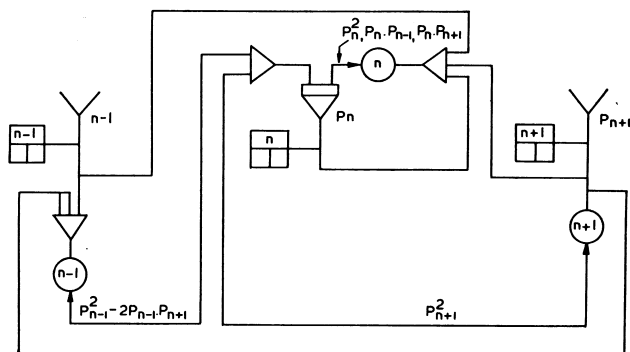


Fig. 2.—Schematic layout of pressure finite-difference point

The part surrounded by a dot/dash line is a device that provides that on the first day there is compression to a preset pressure only, and no decompression to follow it. The reason for this is given in Section 5.2. It functions in the following way. S2 is a latch: it is closed until the run has been commenced and then opened. This latches comparator 4 on the negative side. The comparator is unlatched when the integrator overruns the zero (to throw comparator 0) at the end of day 1. Comparator 4 then receives a positive input and stays on the positive side for the remainder of the run. On the first day then, switch 4B in the F_2 supply is open and there is no decompression. Also switch 4C is on the negative side so that the compression period is controlled by 6B. (6B terminates compression on the first day long before comparator 1.) Potentiometer B at 6A is set to the pressure to be achieved in the first day whilst potentiometer A is set to any value less than the pressure at the end of the first day. Thus when the preset pressure is achieved comparator 6 locks on the positive side.

Various other provisions were made for special runs. A simple one which is shown in **Fig. 3** is switch S1. This was opened after a compression period. The saw-tooth and thus the daily sequence of compression and decompression were interrupted enabling the long-term effects of air leakage and cooling to be studied.

5.2 The operation schedule for the analogue

Preliminary use of the analogue showed that a diurnal cycle of operation would take many weeks to settle down to a condition where the same pressures and temperatures were being achieved each day. However, the drift was not usually important after the third day—the seriousness depending very much on rock properties—and it was decided to use the third day as a normal day on which the standard conditions of pressure and temperature should be achieved.

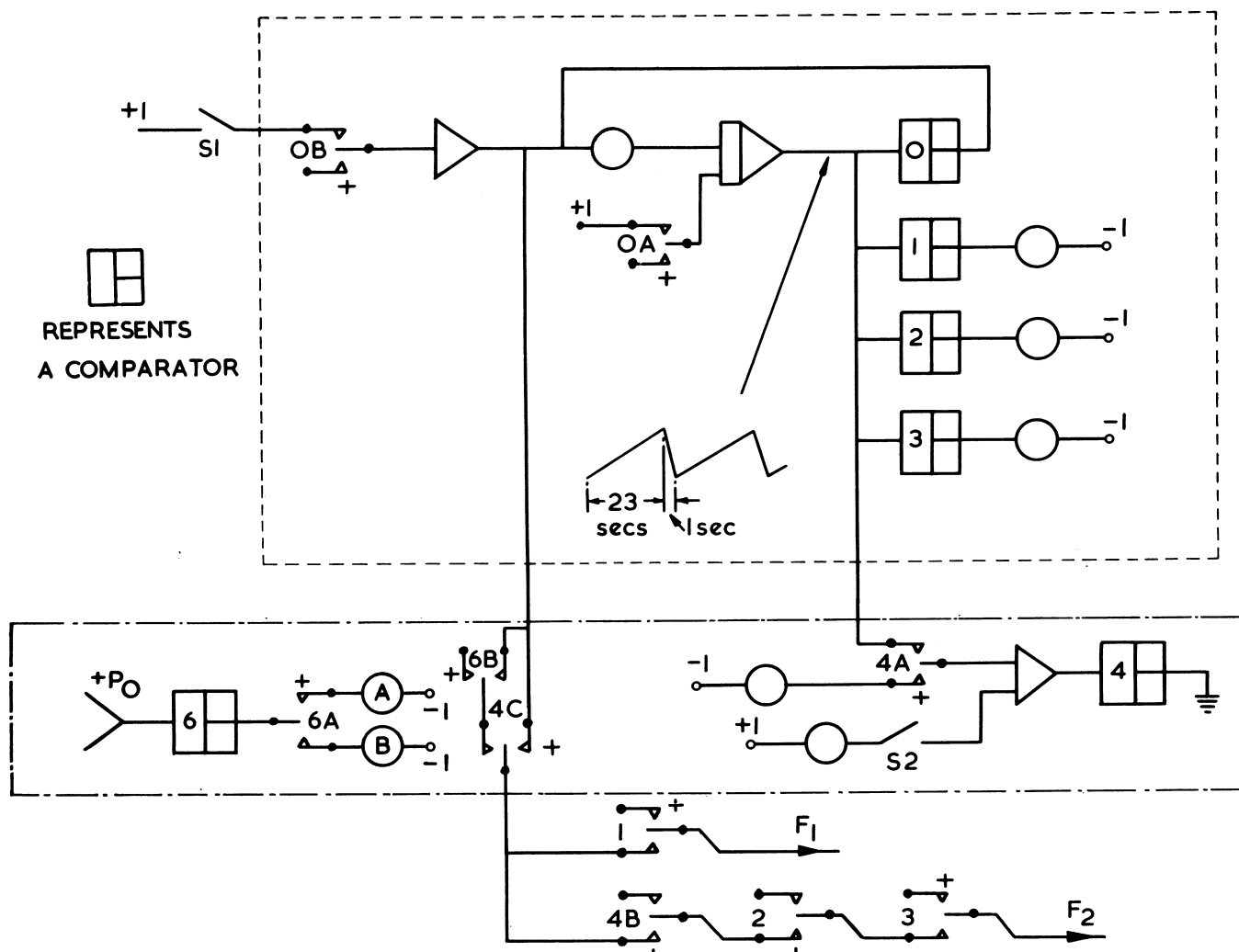


Fig. 3.—The switching system

The computer was time-scaled so that one second on the machine represented one hour of operation of the air storage scheme. This made it necessary to make automatic the switching on and off of the compressor and decompressor, in order to make their timing sufficiently accurate. The automatic switching arrangement meant in turn that the same time schedule of operations had to be used each day.

The fixing of the time schedule was useful, and in fact necessary for the comparison of the effects of variation of the physical parameters of the system. In practice it would not be necessary and where, as in Fig. 5, the maximum pressure rises above 50 atmospheres after the third day, this only means the compressor could be switched off a little earlier after the third day.

It might be suggested that instead of running the analogue to a time schedule, a pressure schedule be used. This method is possible, and in fact was tried on the computer, but it is not so easy to make the duration of compression and decompression periods for the third

day equal to those specified. It was still necessary to work with the third day because the drift was now in temperature and in compression and decompression periods, instead of in temperature and in pressure.

It was decided that the initial pressurization of the tunnel should take two days for the runs testing rock properties (although the possibility of pumping up to 50 atmospheres in one day was examined). In order to establish a standard sequence of events for the study of rock properties, etc., it was necessary to fix the manner in which the compression to 50 atmospheres was distributed between the first two days. The procedure adopted was to pump up to such a pressure that leakage of air left the tunnel at 18 atmospheres at the beginning of the second day's compression period. Although this is arbitrary it did leave the tunnel air pressure at the sort of value it would have at the beginning of compression when the system was being operated every day. The second day could then go straight into the fixed compression period cycle of running.

The standard compression rate was that required when there was no air leakage. On the analogue it was multiplied by a factor X , according to the adjustment required to pump the tunnel up to 50 atmospheres on the third day.

In most of these runs the time schedule for the second and subsequent days was

- 0–9 hours compression
- 9–11 hours resting
- 11–14 hours decompression
- 14–24 hours resting

Fig. 4 shows the variations of tunnel air pressure, tunnel air temperature, rock surface temperature and air leakage, over a period of three days.

The initial rise to a peak value of tunnel air temperature is not an error in the analogue. To test for this the time behaviour of the analogue was slowed down first by a factor of four then a further factor of ten, making a factor of forty in all. The peak was unchanged and so was not due to slow response of servo mechanism in the computer. The initial temperature of the cave air was raised to the value that the temperature falls to after passing the peak. It still rose to the same peak value showing it was not due to overshooting of servo-mechanisms; slowing the time scale also bore this out. The temperature was also started at the peak value but it still followed the same curve. The early hand calculations carried out also showed this peak, although they were discounted at the time as a truncation error.

The most probable explanation for this peak in air temperature is that the initial mass of air in the tunnel is very low and its density is also low. The density appears in the heat transfer function so making this function low. Under these circumstances the air will heat up very rapidly at a rate not much less than adiabatic. The air density rapidly increases and as the heat transfer function gains importance more heat is lost to the rock. This will be accentuated at first because the $(T - \theta_0)^{4/3}$ term has become so large and a rapid fall of temperature ensues forming the peak.

The slight fall in pressure when the compressor is switched off, and the rise and occasional slight fall when the decompression ends are due to a combination of leakage and cooling (or warming) of the air according to the conditions obtaining.

Fig. 5 shows a typical variation over a period of six days of rock temperature and pressure at a number of their respective finite-difference points. The closeness of the temperature and pressure curves in Fig. 5 depends on thermal and gaseous diffusivity of the rock.

The runs list consisted of numerous permutations of the various rock properties, tunnel dimensions, and compression and decompression schedules. These were analyzed by picking out data of particular interest and cross-plotting against the properties being varied. From the considerable number of graphs obtained a few are

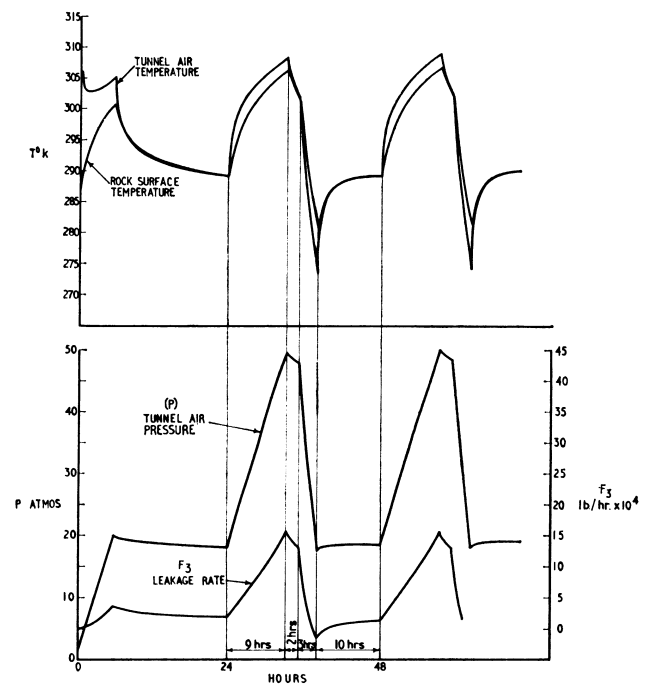


Fig. 4.—Three-day run under standard conditions

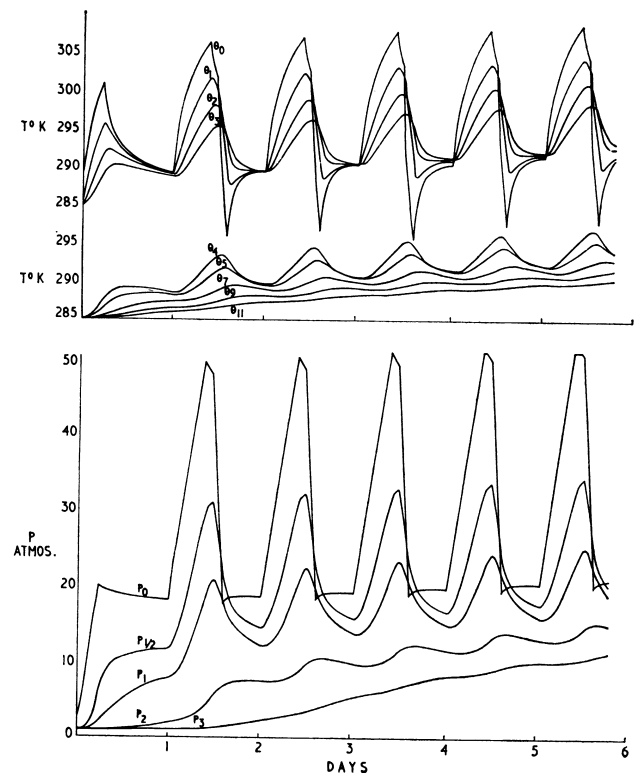


Fig. 5.—Six-day run showing temperatures and pressures at their respective finite-difference points in the rock.

reproduced in Fig. 6. They illustrate the behaviour of some of the more important parameters. The tunnel air temperature is important because it decides how large a tunnel is required to store a certain mass of air. The air leakage is important for its direct effect on the economics of the whole project. The tunnel radius influences the air leakage and so is important for the same reason.

6. Conclusions

Amongst the profusion of data the following are probably the most important.

- The crucial parameters controlling feasibility and machinery specifications for the scheme are rock porosity and permeability. They are also the properties about which least is known.
- The maximum tunnel air temperature depends on thermal properties of the rock and so the latter influence the tunnel volume required for a given size of generating station.
- The variation of tunnel radius has a marked effect on tunnel air temperature and air leakage.

Notation

$\frac{dQ}{dt}$ = rate of heat flow to the rock (CHU.hr⁻¹).

B = constant of proportionality.

$k_a = k_a(T)$ = thermal conductivity of air (CHU.hr⁻¹ ft⁻² °C⁻¹ ft).

T = mean air temperature of the tunnel air (°K).

θ_0 = rock surface temperature (°K).

ρ_a = air density in tunnel (lb. ft⁻³).

$\mu_a = \mu_a(T)$ = air viscosity in tunnel (lb.ft⁻¹hr⁻¹).

r_0 = tunnel radius (ft.).

l = total length of tunnel (ft.).

δW = the work done by the system (CHU).

dU = the increase of internal energy of the system (CHU).

δQ = the heat leaving the system (CHU).

dm = an element of mass entering the system (lb.).

m = the total air mass in the tunnel (lb.).

h = specific enthalpy (CHU.lb⁻¹).

R = gas constant (CHU.°K⁻¹lb⁻¹).

C_v = specific heat at const. volume (CHU.lb⁻¹ °K⁻¹).

u = specific internal energy (CHU.lb⁻¹).

m = mass of gas in tunnel (lb.).

F_3 = leakage rate.

F_1 = compression rate.

F_2 = decompression rate.

θ = rock temperature (°K).

r = distance from tunnel axis (ft.).

k_r = thermal conductivity of the rock.

ρ_r = density of the rock (CHU. °K⁻¹ ft⁻³ hr⁻¹) (lb. ft⁻³).

c_r = specific heat of rock (CHU.lb⁻¹ °K⁻¹).

P = air pressure (atmospheres).

V = tunnel volume (ft³).

m = mass of air in tunnel (lbs.).

R = gas constant (atm. ft³ lb⁻¹ °K⁻¹).

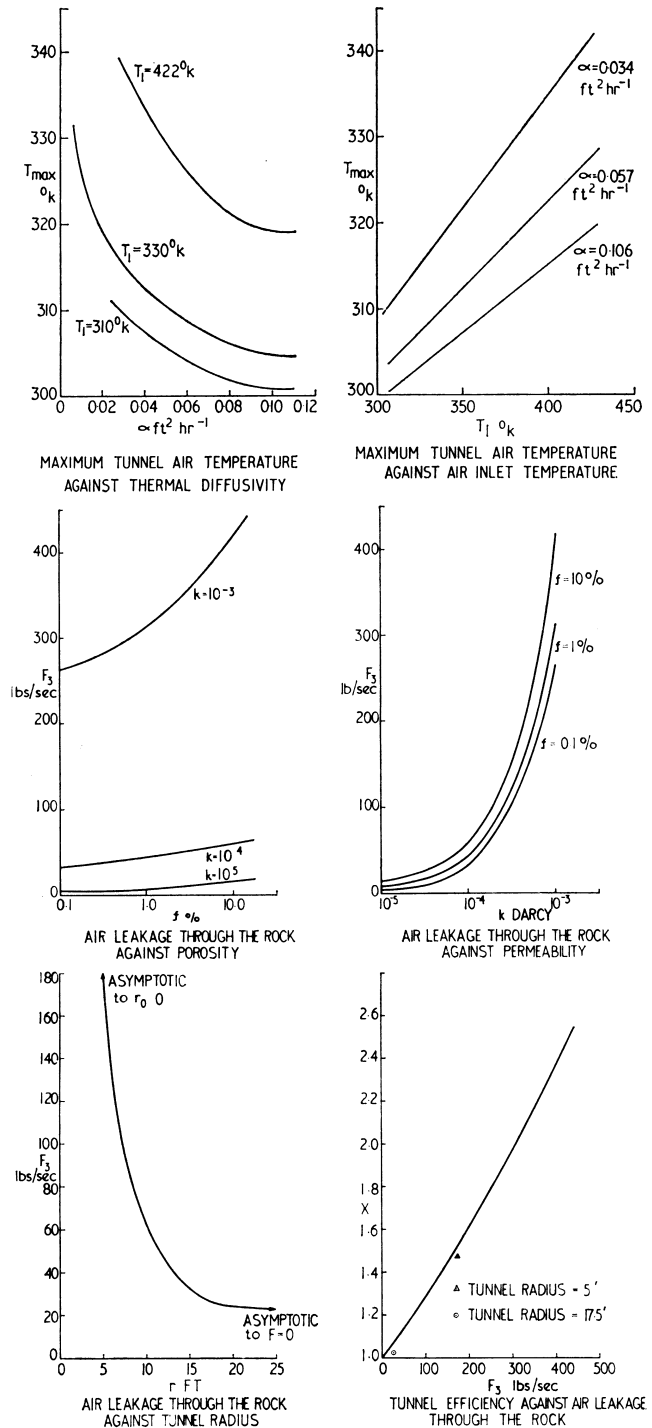


Fig. 6.—Some representative results obtained by cross-plotting data from the computer

f = porosity (dimensionless).

μ = viscosity (centipoises).

k = permeability (darcys).

c = dimension correcting factor.

$\left(\frac{\rho}{P}\right)$ = air density at pressure P in atmos. and therefore takes the value of ρ at 1 atmos.

- c' = dimension correcting factor.
 dH = the enthalpy transported into the system across its boundary (CHU).
 l' = the effective tunnel length considering the tunnel to be a single long straight one. (This was taken as the total tunnel length for the purpose of the computation.)

Acknowledgements

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Correspondence

To the Editor,
The Computer Journal.

Sir,

Since its publication by Rosenbrock (1960), the function $Q = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$ has become a classic for testing various minimization methods, as indicated in papers by Powell (1965) and Nelder and Mead (1965), where 70 and 150 evaluations of Q are required for convergence.

It is curious that, so far as I can discover, the most fundamental application of least-squares techniques has not been mentioned. To this end, the function was formulated as a least-squares problem with the parameters and submitted to the Los Alamos Least Squares Program of Moore and Zeigler (1959). The following table of results, given to eight figures for those who may wish to make a comparison, shows that the venerable Gauss linearization technique (applied with no attempt at step-size optimization) gives an exact solution with only four evaluations of Q . The program, originally written in FORTRAN II, is now available in FORTRAN IV and was run on an IBM 7094.

Table

Iteration			
1	-1.2000000×10^0	1.0000000×10^0	2.4199999×10^1
2	9.9999548×10^{-1}	-3.8399892×10^0	2.3425407×10^3
3	1.0000039×10^0	1.0000077×10^0	$1.4983792 \times 10^{-11}$
4	1.0000000	1.0000000	0

It is to be noted that the behaviour of Q is hardly monotonic. It seems to me that too much is staked on the requirement of monotonicity for a minimization procedure to be good and desirable. Some procedures that require the decrease of the object function at each step have been found at the Los Alamos Scientific Laboratory to "bog down" far short of their optimal value. Thus, the monotonic decrease is neither a necessary nor a sufficient condition that a particular method be satisfactory.

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Yours truly,

ROGER H. MOORE.

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 9 June 1965.

Dr. Rosenbrock replies.

Many years ago I solved this problem by the Newton-Raphson method, which is almost equally fast. This I regarded as amusing rather than significant, because in an engineering problem it would be entirely a matter of chance if the behaviour of the function at the remote points reached by these methods gave any information about the behaviour in the vicinity of the optimum.

I feel that Dr. Moore has missed the point of the example. This was concocted to have the difficulties with which ordinary hill-climbing methods have to contend in practice, and it was implied that the rules of the game did not allow us to exploit the particular form of the function. After all, if we simply want the answer, the easiest way is to use the calculus. If the example had been devised to show up the difficulties of Newton-Raphson or the Gauss linearization, it would have looked rather different. Dr. Moore might like to try out his method on the function

$$Q = \left(\frac{x^2}{1+x^2} \right)^2 + \left(\frac{e^{10x}}{10} \right)^2$$

starting from $x = -10$.