c' = dimension correcting factor.

dH = the enthalpy transported into the system across its boundary (CHU).

l' = the effective tunnel length considering the tunnel to be a single long straight one. (This was taken as the total tunnel length for the purpose of the computation.)

Acknowledgements

The author wishes to acknowledge the interest and encouragement of Mr. R. A. Laws during the course of this work. The problem originated in a feasibility study carried out by P. J. E. Wraith and E. P. Morris of C.E.G.B. The paper is published by permission of the Central Electricity Generating Board.

Correspondence

To the Editor,
The Computer Journal.

Sir,

Since its publication by Rosenbrock (1960), the function $Q = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$ has become a classic for testing various minimization methods, as indicated in papers by Powell (1965) and Nelder and Mead (1965), where 70 and 150 evaluations of Q are required for convergence.

It is curious that, so far as I can discover, the most fundamental application of least-squares techniques has not been mentioned. To this end, the function was formulated as a least-squares problem with the parameters and submitted to the Los Alamos Least Squares Program of Moore and Zeigler (1959). The following table of results, given to eight figures for those who may wish to make a comparison, shows that the venerable Gauss linearization technique (applied with no attempt at step-size optimization) gives an exact solution with only four evaluations of Q. The program, originally written in FORTRAN II, is now available in FORTRAN IV and was run on an IBM 7094.

Table

Itera-tion			
1	$-1 \cdot 2000000 \times 10^0$	$1 \cdot 0000000 \times 10^0$	$2 \cdot 4199999 \times 10^1$
2	$9 \cdot 9999548 \times 10^{-1}$	$-3 \cdot 8399892 \times 10^0$	$2 \cdot 3425407 \times 10^3$
3	$1 \cdot 0000039 \times 10^0$	$1 \cdot 0000077 \times 10^0$	$1 \cdot 4983792 \times 10^{-11}$
4	$1 \cdot 0000000$	$1 \cdot 0000000$	0

It is to be noted that the behaviour of Q is hardly monotonic. It seems to me that too much is staked on the requirement of monotonicity for a minimization procedure to be good and desirable. Some procedures that require the decrease of the object function at each step have been found at the Los Alamos Scientific Laboratory to "bog down" far short of their optimal value. Thus, the monotonic decrease is neither a necessary nor a sufficient condition that a particular method be satisfactory.

References

Moore, R. H., and Zeigler, R. K. (1959). "The Solution of the General Least Squares Problem with Special Reference to High-speed Computers," Los Alamos Scientific Laboratory Report LA-2367.

Nelder, J. A., and Mead, R. (1965). "A Simplex Method for Function Minimization," *The Computer Journal*, Vol. 7, p. 308.

Powell, M. J. D. (1965). "A Method for Minimizing a Sum of Squares of Non-linear Functions without Calculating Derivatives," *The Computer Journal*, Vol. 7, p. 303.

Rosenbrock, R. H. (1960). "An Automatic Method for Finding the Greatest or Least Value of a Function," *The Computer Journal*, Vol. 3, p. 175.

Yours truly,

Roger H. Moore.

University of California,
Los Alamos Scientific Laboratory,
P.O. Box 1663,
Los Alamos,
New Mexico, U.S.A.
9 June 1965.

Dr. Rosenbrock replies.

Many years ago I solved this problem by the Newton-Raphson method, which is almost equally fast. This I regarded as amusing rather than significant, because in an engineering problem it would be entirely a matter of chance if the behaviour of the function at the remote points reached by these methods gave any information about the behaviour in the vicinity of the optimum.

I feel that Dr. Moore has missed the point of the example. This was concocted to have the difficulties with which ordinary hill-climbing methods have to contend in practice, and it was implied that the rules of the game did not allow us to exploit the particular form of the function. After all, if we simply want the answer, the easiest way is to use the calculus. If the example had been devised to show up the difficulties of Newton-Raphson or the Gauss linearization, it would have looked rather different. Dr. Moore might like to try out his method on the function

$$Q = \left(\frac{x^2}{1 + x^2}\right)^2 + \left(\frac{e^{10x}}{10}\right)^2$$

starting from $x = -10$.