where $$a_s = 2\int_0^1 \frac{x}{12}(2x^2 - x^3 - 1)\sin(s\pi x)dx$$

$$= \frac{4}{s^5\pi^5}(\cos(s\pi) - 1).$$

Computation of the exact solution at the mesh points with spacings $\Delta x = 0{\cdot}05$, $\Delta t = 0{\cdot}00125$ from equation (63) correct to five decimal places was obtained and compared with the results obtained from the finite-difference equations (5). Both short-time and long-time studies of the exact and finite-difference solutions were compared, and agreement to within the derived error bound given by equation (62) was obtained. A typical sample of these comparisons are shown in **Table 1** for $t = 0{\cdot}02$ and $x = 0(0{\cdot}05)\ 0{\cdot}5$, and in **Table 2** for $x = 0{\cdot}5$ and larger values of t.

The stability criteria of (5) was checked experimentally by taking a large variety of mesh sizes Δx and Δt and obtaining the solutions on the Sheffield University Mercury computer. In each case, no evidence of numerical instability due to growth of round-off error was found.

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Book Review

Computing Methods (*Volumes I and II*), by I. S. Berezin and N. P. Zhidkov, 1965; 464 and 679 pages. (Oxford: *Pergamon Press Ltd.*, 100s. per volume.)

This enormous work—1143 pages in all—attempts to cover the whole field of what we would call numerical analysis. The first volume has six chapters, on approximate quantities and errors, interpolation, numerical integration and differentiation, general analytic methods of approximation to functions, least squares approximation: the second, on linear algebraic equations, non-linear algebraic equations and transcendental equations, eigenvalues and vectors of matrices, ordinary differential equations, partial differential equations, and integral equations. I found it all very laborious and uninspiring and I could not detect anything new in either the methods or the results. On the contrary, the treatment has a very old-fashioned air, with pages of heavy algebra, extensive displays of formulae—for example, for numerical integration and differentiation—which could have been put into appendices, or, better, left out altogether, and exhaustive pursuit of details with no great reward in the end, as in the 40 pages on the Runge-Kutta method. The contrast with the elegant writings of Henrici, for example, is very striking. The point of view is wholly that of the hand computer (there is a bare mention of electronic machines in the introduction to Volume I, which includes the statement that a modern high-speed computer operates at about 8,000 instructions per second). Even so, I got a strong impression all the way through that the authors had never done much actual computation, except perhaps a few academic exercises, and this was reinforced by particular points of detail. Thus the account of the Euler-Maclaurin formula relating an integral to a series does not say that the series is asymptotic, not convergent; and the brief note on page 294 on calculation of integrals with a variable upper limit suggests that they have never heard of the neat and efficient methods due I think to Comrie and given years ago in *Interpolation and Allied Tables*.

There is of course a great deal of information in the book, especially in the chapters on approximation; but for a work published in 1965 the omissions are unforgiveable. On quadrature and ordinary differential equations there is no mention of the modern European school—Dahlquist, Henrici, Rutishauser, Stiefel; on matrix calculations, Wilkinson's name appears only in a 1948 reference, and the methods of Jacobi, Givens and Householder for eigenvalue calculations are not mentioned; nor is the work of Young, Richtmyer, Varga on partial differential equations, nor indeed any of the modern American writers. The publishers should have given the date of the original Russian edition. The references, most of which are to work published between 1948 and 1954 with none later than 1958, suggest that it was written in 1955–56; if that is so, one can understand why the important modern work has been missed. It strikes me as yet another warning against trying to write everything down in one great work.

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