Theorem 5

Let Sg be a finite set of ground clauses not containing the empty clause. If for every renaming of its literals Sg retains at least one positive clause then it is unsatisfiable. This also holds if we substitute "negative" for "positive".

Proof:

Suppose Sg were satisfiable. Then Sg has a model M. Some of the atoms in M will in general be negated and some not. Let us now apply a renaming which converts all the un-negated atoms in M into negated ones, thus transforming the set Sg into the renamed set S_g', say. Since S_g' has a model consisting entirely of negative literals, every clause of S_g' must have at least one negative literal.

Hence we have the result that if S_g is satisfiable there is a renaming under which no clause is positive. From this it follows that if in every renaming some clause is positive, then Sg is unsatisfiable.

The second part of the theorem can be proved by applying a renaming which converts all the negated atoms of M into un-negated ones.

It is instructive to note why when this theorem holds for a set Sg of ground clauses it does not necessarily hold for a set S of clauses from which Sg has been derived by instantiation. Consider the following example, in which S consists of a single clause and Sg of two:

$$S = \{P(x, a) \vee Q(y, f(y))\}$$

$$Sg = \{P(a, a) \vee Q(a, f(a))\}$$

$$\& \{P(f(a), a) \vee Q(a, f(a))\}$$

We see that while some of the renamings of Sg correspond to renamings of S,
e.g. $P(a, a)$ renamed $\bar{P}'(a, a)$ and $P(f(a), a)$ renamed $\bar{P}'(f(a), a)$, others do not,
e.g. only $P(a, a)$ renamed $\bar{P}'(a, a)$.

In fact (if for convenience we treat identity as a renaming too) S has $2^2 = 4$ renamings, while Sg has $2^3 = 8$, and only 4 of the latter arise from the former.

Thus, in general, all possible renamings of a set S of clauses, do not, by instantiation, generate all possible renamings of a set of ground clauses Sg derived from it, and for this reason Theorem 5 cannot be taken over to sets of non-ground clauses.

Since, however, Theorem 4 obviously applies to ground clauses too, sets of ground clauses can be tested definitively for satisfiability by renaming.

The possibility of a useful proof procedure arises if one could design an algorithm, which—working directly on a set S of clauses—would determine the effects of renaming for all possible substitution instances, rather in the way Robinson's unification algorithm (Robinson, 1965) effects all possible matchings of instances of clauses without actually explicitly generating the instances.

References

Davis, M. (1963). "Eliminating the irrelevant from mechanical proofs," *Proceedings of Symposia in Applied Mathematics*, American Mathematical Society, p. 15.

Robinson, J. A. (1965). "A machine-oriented logic based on the resolution principle," *Journal of the Association for Computing Machinery*, Vol. 12, p. 23.

Note added in proof: Robinson's results on P_1-deduction will be found in a forthcoming article in the *International Journal of Computer Mathematics*.

Book Review

Journal of Differential Equations, *Volume* 1, *Number* 1, edited by J. P. La Salle, 1965; 113 pages. (New York and London: *Academic Press*, \$9 annually).

The first paper (26 pages) by H. W. Knobloch is on comparison theorems for nonlinear second order differential equations. This is more difficult than the attractive Sturm's theory on comparison and oscillation theorems for linear second order differential equations, but some results are obtained and the theory is applied to derive properties of the trajectory of solutions of Van der Pol's equation, of the form $y'' = f(y)y' - y$ in the (y, y') plane.

The second paper (12 pages) by George Hufford is on the characteristic matrix of a matrix of differential operators. A system of n linear partial differential equations in n dependent variables and several independent variables leads to the study of the n by n matrix $A(D)$ whose elements $a_{ij}(D)$ are polynomials in D, the vector whose components are the partial differentiation operators with respect to the independent variables. The principal part $h_{ij}(D)$ of $a_{ij}(D)$ is the homogeneous polynomial consisting of the terms of highest order in $a_{ij}(D)$. The principal parts form the characteristic matrix $H(D)$. This paper deals with various definitions of H and its invariants under linear transformations of the dependent variables.

The third paper (56 pages) by Kenneth L. Cooke is on the condition of regular degeneration for singularly perturbed linear differential-difference equations of the form $eA(e)u + Bu = v$ where A, B are difference-differential operators, e is the perturbation parameter, and the order of A is higher than the order of B. The behaviour of the perturbation leads to a study of the perturbation of zeros of exponential polynomials.

The last paper (19 pages) by Paul Fife is on exterior Dirichlet problems and an application to boundary layer theory. This deals with parabolic partial differential equations where the solution is periodic in t, the boundary conditions are specified on the boundary of a domain in the space variables, and the solution is required outside this domain. Solutions which remain bounded or tend to a limit as the space variables tend to infinity are investigated, and the results are applied to the Navier-Stokes and Prandtl problems.

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