Edwards, A. W. F. and Cavalli-Sforza, L. L. (1964). Reconstruction of Evolutionary Trees. Pp. 67–76 in *Phenetic and Phylogenetic Classification*. Systematics Association Publication No. 6. (London: Systematics Association, xi + 164 pp. illust.).

Fry, W. G. (1964). "The Pycnogonida and the Coding of Biological Information for Numerical Taxonomy." *Systematic Zoology*, Vol. 13, pp. 32–41.

Hodson, F. R., Sneath, P. H. A. and Doran, J. E. A study of bronze fibulae from the La Tène culture (to be published).

Rose, M. J. (1964). Classification of a Set of Elements, *Computer Journal*, Vol. 7, pp. 208–211.

Sokal, R. R. and Sneath, P. H. A. (1963). *Principles of Numerical Taxonomy.* (San Francisco and London: W. H. Freeman, xvi + 359 pp. illust.).

Correspondence

To the Editor,
The Computer Journal.

Nonlinear programming test problems

Sir,

In a recent article by M. J. Box* on constrained optimization there is included a small five-variable nonlinear programming problem which is described as a very difficult problem. Since it is one of the few examples of a nonlinear programming problem given in the literature, it is likely to be used as a test problem by other persons working in the area of nonlinear programming. Persons using it as a test problem ought to be aware that the problem is easily converted to a linear programming problem. The form of this problem given in Box's article obscures the fact that this can be done by a simple transformation. It was in a discussion of this problem with Professor C. E. Lemke, of Rensselaer Polytechnic Institute, that we noticed this simple transformation.

The problem can be rewritten as follows:

$$\min_x \left\{ f(x) = b_0 + a_{01}x_1 + \left(\sum_{j=2}^{5} a_{0j}x_j \right) x_1 \right\}$$

subject to

$$0 \leqslant a_{i1}x_1 + \left(\sum_{j=2}^{5} a_{ij}x_j \right) x_1 \leqslant b_i \quad i = 1, 2, 3$$

$$x_1 \geqslant 0,\ 1 \cdot 2 \leqslant x_2 \leqslant 2 \cdot 4,\ 20 \cdot 0 \leqslant x_3 \leqslant 60$$

$$9 \cdot 0 \leqslant x_4 \leqslant 9 \cdot 3,\ 6 \cdot 5 \leqslant x_5 \leqslant 7 \cdot 0.$$

Then by letting $y_i = x_1 x_i$, $i = 2, 3, 4, 5$ and $y_1 = x_1$ we obtain the following linear programming problem:

$$\min_y \left\{ g(y) = b_0 + \sum_{j=1}^{5} a_{0j}y_j \right\}$$

$$0 \leqslant \sum_{j=1}^{5} a_{ij}y_j \leqslant b_i \quad i = 1, 2, 3$$

$$y_i \geqslant 0 \ i = 1, 5$$

$$y_2 - 1 \cdot 2\, y_1 \geqslant 0, \qquad 2 \cdot 4\, y_1 - y_2 \geqslant 0$$

$$y_3 - 20 \cdot 0\, y_1 \geqslant 0, \qquad 60 \cdot 0\, y_1 - y_3 \geqslant 0$$

$$y_4 - 9 \cdot 0\, y_1 \geqslant 0, \qquad 9 \cdot 3\, y_1 - y_4 \geqslant 0$$

$$y_5 - 6 \cdot 5\, y_1 \geqslant 0, \qquad 7 \cdot 0\, y_1 - y_5 \geqslant 0.$$

I solved this problem on the IBM 7040 using linear programming routine LP-40, and obtained the following optimal solution after six simplex iterations:

$$g = -5{,}280{,}344 \cdot 9$$
$$y_1 = 4 \cdot 53743,\ y_2 = 10 \cdot 88983,\ y_3 = 272 \cdot 24584$$
$$y_4 = 42 \cdot 19811,\ y_5 = 31 \cdot 76202$$

which in terms of the original variables gives $f = -5{,}280{,}344 \cdot 9$

$$x_1 = 4 \cdot 53743,\ x_2 = 2 \cdot 40000,\ x_3 = 60 \cdot 00000$$
$$x_4 = 9 \cdot 30000,\ x_5 = 7 \cdot 00000.$$

It is interesting to note that the original nonlinear problem is a non-convex problem; the feasible region is not a convex region nor is the objective function convex. Yet the transformation results in a convex, in this case linear, programming problem.

The values of the constants in this form are

$a_{01} = -8{,}720{,}288 \cdot 795$	$a_{21} = -155{,}011 \cdot 1055$
$a_{02} = -150{,}512 \cdot 524$	$a_{22} = 4{,}360 \cdot 5334$
$a_{03} = -156 \cdot 695$	$a_{23} = 12 \cdot 9492$
$a_{04} = -476{,}470 \cdot 319$	$a_{24} = 10{,}236 \cdot 8839$
$a_{05} = -729{,}482 \cdot 825$	$a_{25} = 13{,}176 \cdot 7859$
$a_{11} = -145{,}421 \cdot 4004$	$a_{31} = -326{,}669 \cdot 5059$
$a_{12} = 2{,}931 \cdot 1506$	$a_{32} = 7{,}390 \cdot 6840$
$a_{13} = -40 \cdot 4279$	$a_{33} = -27 \cdot 8987$
$a_{14} = 5{,}106 \cdot 1920$	$a_{34} = 16{,}643 \cdot 0759$
$a_{15} = 15{,}711 \cdot 3600$	$a_{35} = 30{,}988 \cdot 1459$
$b_0 = -24{,}345 \cdot 0$	$b_2 = 294{,}000 \cdot 0$
$b_1 = 294{,}000 \cdot 0$	$b_3 = 277{,}200 \cdot 0$

Sincerely yours,

W. Charles Mylander

Advanced Research Division,
Research Analysis Corp.,
McLean, Virginia, U.S.A.
3 August, 1965.

* Box, M. J. (1965). "A new method of constrained optimization and a comparison with other methods," *The Computer Journal*, Vol. 8, p. 42.

Downloaded from https://academic.oup.com/comjnl/article/8/4/391/400883 by guest on 19 April 2024