Table 1

The value and error of the solution at $x = 0 \cdot 2$ and $0 \cdot 5$

VALUE OF h	SOLUTION AT $0 \cdot 2$	ERROR AT $0 \cdot 2$	SOLUTION AT $0 \cdot 5$	ERROR AT $0 \cdot 5$
0·1	0·67621 94	−0·00000 47	0·27331 2	−0·00001 4
0·05	0·67622 38	−0·00000 03	0·27332 56	−0·00000 09
0·025	0·67622 41	0	0·27332 65	−0·00000 005

There seems to be an obvious gain in using this method rather than a second order method, such as

$$y_{i+1} - 2y_i + y_{i-1} + h^2 f(x_i, y_i, (y_{i+1} - y_{i-1})/2h) = O(h^4), \text{ for } i = 1, \ldots, n - 1.$$

In our method we have obtained an error of $O(h^4)$ instead of $O(h^2)$ with only four times as much work. h^2 extrapolation could indeed be used on second order methods but we could use h^4 extrapolation on our own methods, and it would seem that the nearer we started to the true solution before we introduced the instability possibilities of extrapolation the better. In comparison with explicit methods of solving the problem the method seems less efficient. A Runge–Kutta method for instance which obtains the same order of accuracy needs only four function evaluations per step. We usually need to evaluate the recurrence relation twice to find y_{i+1} with sufficient accuarcy, and thus need eight function evaluations per step. There may, however, be circumstances when explicit methods lead to unnecessary instabilities and where the present method might be of value.

Acknowledgements

Dr. E. T. Goodwin derived the specific form of the error in the approximation (2). I would like to thank him for this as well as for further help in presentation.

This research was carried out while I was the holder of the I.C.T. research scholarship. I would like to thank International Computers and Tabulators, Australia Ltd., for their kind and generous support.

I would also like to thank my supervisor, Professor J.M. Bennett, for his encouragement and for the facilities of the Basser Computing Laboratory, School of Physics, Sydney University.

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Book Review

Sampling Systems Theory and its Application, Volumes 1 *and* 2, by Ya. Z. Tsypkin, 1964; 742 pages. (Oxford: *Pergamon Press Ltd.*, 100s. per volume).

The author of these two volumes works at the *Academy of Sciences* of the USSR and is an acknowledged expert in this field, having published several papers in recent years. The two volumes are entirely complementary and require no prior knowledge of sampling systems.

The first chapter classifies various pulse systems, and gives practical examples. It is possible that benefit could have been gained if a little of the fine detail could have been sacrificed for a discussion in a broader context of fields where pulse systems are inevitable and, in fields where this is not so, why they may be preferred to continuous systems. Chapter II lays the basic theoretical groundwork and provides a valuable reference provided one does not object to unfamiliar terminology, *e.g.* the use throughout of the discrete Laplace transformation rather than the Z-transform. This chapter is very thorough, although it would have been of advantage if the important results could have been summarized more effectively. Application of the techniques to solving difference equations is included. Chapter III is concerned with the application of the theory to open-loop systems.

Chapter V, in the second volume, considers closed-loop systems and again covers a lot of ground. It is here that some of the author's original work is contained and, therefore, this section is likely to be of most interest to those already proficient in the subject.

Much of the value of these volumes stems from the many wide and varied problems on both closed- and open-loop pulse systems. The bibliography and tables contained in the appendix are also very useful. Of particular interest to those associated with digital computers is their use in control systems; this receives adequate attention.

To summarize, these two volumes certainly merit attention from scientists specializing in sampled-data systems.

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