3. Example (See Nielsen, 1956)

$$y' = \frac{2x-1}{x^2}y + 1 \qquad \begin{cases} x_0 = 1 \\ y_0 = 2 \end{cases} \tag{3.1}$$

$h = 0 \cdot 1$

n	x	y	$T \times 10^{10}$	g
0	1·0	2·0000 0000		1·0600
1	1·1	2·3148 5619	−80482	0·9392
2	1·2	2·6589 3596	−36009	0·8409
3	1·3	3·0317 3475	−15908	0·7650
4	1·4	3·4328 9892	−8946	0·7039
5	1·5	3·8622 0178	−4804	0·6532
6	1·6	4·3194 6504	−3175	0·6101
7	1·7	4·8045 7482	−1675	0·5729
8	1·8	5·3174 3008	−1346	0·5403
9	1·9	5·8579 6919	−627	0·5115
10	2·0	6·4261 2946	−668	0·4858
11	2·1	7·0218 7604	−226	0·4627
12	2·2	7·6451 6696	−391	0·4417

The exact solution is $y = x^2\left(1 + \exp\left(\frac{1}{x} - 1\right)\right)$. (3.2)

Substitution in (1.4) gives $E_{12} = 14{,}100 \times 10^{-8}$ (3.3)

Approximating T_n by $-146 \times 10^{-7} \times \left(\frac{1}{2}\right)^n$ and assuming that $e_0 = e_1 = 0$, the values of e_2, e_3 and e_4 calculated from (2.3) are found to be

$$\left.\begin{aligned} e_2 &= 75{,}105 \times 10^{-10} \\ e_3 &= 49{,}096 \times 10^{-10} \\ e_4 &= 102{,}568 \times 10^{-10}. \end{aligned}\right\} \tag{3.4}$$

Putting $n = 2$ and 4 in (2.25) and equating to e_2 and e_4 in (3.4) we find that

$$\left.\begin{aligned} A_0 &= 2{,}987 \times 10^{-8} \\ B_0 &= -\,3{,}412 \times 10^{-8}. \end{aligned}\right\} \tag{3.5}$$

Substituting for A_0 and B_0 in (2.25) with $n = 12$ we obtain

$$e_{12} = 3{,}762 \times 10^{-8}. \tag{3.6}$$

The value of (3.2) when $x = 2 \cdot 2$ is 7·6451 5888.
The actual error is thus 808×10^{-8}.

References

Milne, W. E. (1953). *Numerical Solution of Differential Equations*, Wiley: New York, pp. 66–69.

Nielsen, K. L. (1956). *Methods in Numerical Analysis*, Macmillan: New York, pp. 230–232.

Book Review

Optimization Theory and the Design of Feedback Control Systems, by C. W. Merriam III, 1964; 390 pages. (Maidenhead: *McGraw-Hill Publishing Company Ltd.*, 112s.).

This book, by a well-known figure in both academic and industrial research into advanced control problems, is a thorough introduction to Optimal Control Theory as applied to continuous dynamic systems. Its clear exposition starts with the simplest problem, that of choosing the best parameters for a fixed controller, and continues through optimum linear, and linear optimum systems to the design of non-linear control systems.

Full explanations of each point are given, aided by simple worked examples, with illustrated solutions. Included is an account of the application of variational calculus, dynamic programming, and the "maximum principle" to control problems, showing the relation and the differences between the three methods. There is an enlightening chapter on the numerical solution of the two-point boundary problem.

The appendices, apart from a summary of the extensive basic notations and a survey of relevant literature, include short articles on the essential aspects of random signal theory, and on the implications of the state vector description of dynamic systems. There is a fully worked-out example of a linear optimum control system for the aircraft landing problem, and notes on computer methods of numerical integration of differential equations and on the approximation of the Hamiltonian function. Not least is a set of problems based on the content of each chapter.

Based on a post-graduate lecture course, and especially suitable as an advance text, the book will also prove valuable to the practising engineer and mathematician who must turn optimal theory into working control schemes. Such applications will provide employment for man and computer for a good many years—indeed as long as there are processes to control, and computers with which to do so.

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