Computing the state of the economy

By Lucy Joan Slater*

This paper contains an outline of a computer program developed in the Department of Applied Economics of Cambridge University. This program represents an outline model of the British economic system, which is simple enough to permit of actual numerical realization.

About five years ago a group of econometricians, working under the general direction of Professor Richard Stone, obtained a grant from the Ford foundation, to investigate the problems of stimulating growth in the British economy. The first aim of the group was to produce a prototype model of the British economic system, which would at once be simple enough to permit of actual numerical calculations within its framework, and yet be realistic enough to give opportunities of experimenting with various alternative structural hypotheses.

Since the model was intended to provide a prototype computing engine for the economy similar to Stevenson's steam engine, the project was code-named the *Rocket project*.

Several members of the group started the onerous task of collecting and building up the initial data required, while Professor Stone, J. A. C. Brown and the author worked out the mathematical framework. Over several years we developed the general model which is the subject of this paper. Most of the early experiments were carried out on the EDSAC 2 computer at the Cambridge University Mathematical Laboratory by kind permission of its director Professor M. V. Wilkes. Recently the model has been reprogrammed for several other British computers.

The source of the data

The original basis of the input data is a highly aggregated Social Accounting Matrix for the British economy, code-named SAM. This was built up from statistics extracted mainly from Government and commercial sources. The first version was based on the year 1954.

In theory a full SAM would have 253 rows and columns, that is some 64,000 items. A matrix of this size would strain the high-speed storage capacity even of the Titan computer, quite apart from the difficulty of amassing the actual items of data. It was decided to restrict our activities to 52 submatrices, each of about size 31 × 31. There is no magic in the number 31. It was chosen because, at the time of our earliest experiments, EDSAC 2 could hold only one matrix of this size at a time.

The 1954 SAM data was used in the development of the original experimental work, to produce reasonably accurate estimates for 1960. These estimates were checked against the actual 1960 figures and any serious disagreements led to adjustments of the mathematical structure. The calculations were then repeated using a 1960 SAM to produce estimates for 1970. We still have some time to wait in order that we may check these results against reality.

The input data

In this paper, a capital letter A will denote a matrix, a small letter $\hat{a}$ will denote a diagonal matrix, a small letter a will denote a vector, and Greek letters such as α, will denote scalar constants.

At present the input data consists of nine constants, thirty-two vectors, and ten matrices, which are listed below. As explained above, it was decided initially to split the economy into thirty-one different sectors. Later versions of the program have provision for fifty sectors. Each Δ item represents the change in the vector over one year. The superfixes represent the position of the vector or matrix in the original SAM matrix.

The notation is that used throughout the Department's series of publications on the project.

Variable	Annual rate of change	Description
Nine input constants		
α		An integer $0 < \alpha < 100$ defining $\alpha\%$, the level of purchase tax.
μ	$\Delta\mu$	The total per head private consumption expenditure.
π	$\Delta\pi$	The ratio of population in the given year to the base year.
β	$\Delta\beta$	The balance of payments (excess of exports over imports, positive we hope).
$i't^{3.15}$	$i'\Delta t^{3.15}$	Foreign tourists' expenditure in U.K.
Thirty-two vectors		
$p_0^{1.3}$		Prices of private current consumption.
$p_0^{1.10}$		Prices of replacement of consumer durables.
$p_0^{1.11}$		Prices of extensions of consumer durables.
$a^{1.9}$		Purchase taxes on industrial extensions.

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$a^{1.3}$		Purchase taxes on consumer current expenditure.
$a^{1.10}$		Purchase taxes on replacement of consumer durables.
$a^{1.11}$		Purchase taxes on extensions of consumer durables.
$a^{1.8}$		Purchase taxes on industrial replacements.
b	Δb	Marginal propensity to spend.
c	Δc	Committed expenditure.
t^8	Δt^8	Investment to replace industrial fixed assets.
t^4	Δt^4	Total public consumption.
t^{12}	Δt^{12}	Investment to replace government social capital.
t^{13}	Δt^{13}	Extensions to stock of social capital.
$t^{1.15}$	$\Delta t^{1.15}$	Exports.
$a^{15.3}$		Imports of current complementary consumers' goods.
$a^{15.4}$		Imports of current complementary government goods.
a_1		Competitive import coefficients.
a_2		Marginal propensities for competitive imports.
a_3		Industrial import coefficients.
k_0		Ratios of fixed assets to commodity outputs.
$a^{1.7}$		Stock/output ratios.
$a^{5.1}$		Customs duties on competitive imports.
$a^{15.7}$		Industrial imports to stock ratios.
Δa_1		The change in imports to stock ratios.

Ten input matrices

$A_0^{1.9}$	Industrial extensions converter.
$A_0^{1.3}$	Consumers' current converter.
$A_0^{1.10}$	Consumers' durable replacement converter.
$A_0^{1.11}$	Consumers' durable extensions converter.
$A_0^{1.8}$	Industrial replacement converter.
$A^{1.4}$	Government current converter.
$A^{1.12}$	Government replacement converter.
$A^{1.13}$	Government extensions converter.
$A^{1.1}$	Commodity/commodity input–output matrix.
C_0	Mix matrix.

Extensive preliminary calculations went into the production of the latest versions of each of these matrices and vectors.

The output results

These consist of eight calculated constants, thirty-one calculated vectors, and two matrices.

Eight calculated constants

$i't^{15.2}$		Total industrial imports.
$i't^{15.7}$		Total industrial imports to stock.
$i't^{15.3} + i't^{15.4}$	$i'\Delta t^{15.3} + i'\Delta t^{15.4}$	Total final complementary imports.
$i't^{15.1}$	$i'\Delta t^{15.1}$	Competitive imports.
$i't^{1.15} + i't^{3.15}$	$i'\Delta t^{1.15} + i'\Delta t^{3.15}$	Total foreign sales of commodities.

Thirty-one calculated vectors

t^3	Δt^3	Total private consumption.
t^{10}	Δt^{10}	Investment to replace consumer durables.
t^{11}	Δt^{11}	Investment to extend consumer durables.
t^8	Δt^8	Investment to replace industrial fixed assets.
$t^{15.3}$	$\Delta t^{15.3}$	Imports of finished products.
$t^{15.4}$	$\Delta t^{15.4}$	Imports of government goods and services.
f^*	Δf^*	Exogenous final output.
$t^{1.7}$		Final demand for capital commodities.
$t^{1.9}$		Final demand for industrial capital extension.
$t^{15.1}$	$\Delta t^{15.1}$	Competitive imports.
$t^{15.2}$	$\Delta t^{15.2}$	Industrial imports.
$t^{15.7}$	$\Delta t^{15.7}$	Capital commodity imports.
$t^{5.1}$	$\Delta t^{5.1}$	Indirect taxes.
t^2	Δt^2	Total industrial current output.
t^9		Total capital extension in industry.
$(T^{2.1})'i$	$(\Delta T^{2.1})'i$	Total commodity output.
f	Δf	Final demand.

The calculations

The calculations required to produce the above results from the given input fall into three natural stages, and one subroutine.

Stage 1 calculates the expenditure e_1 and Δe_1.

Stage 2 deduces the exogenous final output f^* and its change in one year Δf^*.

Subroutine 1 contains a fast matrix inversion subroutine, for matrices with column sums less than unity.

Stage 3 is a two-part iterative process, in two stages. Part 3A calculates a first approximation to the change in total commodity output $(\Delta T^{2.1})'i$. Part 3B takes this first approximation as the starting point and iterates round a cycle until the change produced in $(\Delta T^{2.1})'i$ in one year is negligible. Part 3C forms a first approximation to $(T^{2.1})'i$ the total commodity output; Part 3D repeats the iterative process until convergence is reached, and a stable value of $(T^{2.1})'i$ is found.

Stage 1

This stage of the program reads and checks all the data. Then it takes account of the effects produced by a change in the level of purchase tax α, that is it revises the five "A" matrices, $A_0^{1\cdot 9}$ to $A_0^{1\cdot 8}$ by multiplying each column j by $1/(1 - \alpha a_j)$, from the vectors $a^{1\cdot 9}$ to $a^{1\cdot 8}$.

It revises the three price vectors $p_0^{1\cdot 3}$, $p_0^{1\cdot 10}$ and $p_0^{1\cdot 11}$, by multiplying the jth element of each by

$$(1 - \alpha a_j^{1\cdot 3}), (1 - \alpha a_j^{1\cdot 10}) \text{ and } (1 - \alpha a_j^{1\cdot 11}),$$

respectively, to produce three new price vectors $p_1^{1\cdot 3}$, $p_1^{1\cdot 10}$, and $p_1^{1\cdot 11}$. It stores

$$\Delta p_1 = \{p_1^{1\cdot 3} - p_0^{1\cdot 3}, p_1^{1\cdot 10} - p_0^{1\cdot 10}, p_1^{1\cdot 11} - p_0^{1\cdot 11}\}$$

and

$$p_1 = \{p_1^{1\cdot 3}, p_1^{1\cdot 10}, p_1^{1\cdot 11}\}.$$

This is the adjusted vector of current prices. It forms the expenditure vector

$$e_0 = \pi \hat{p}_1^{-1}\{b\mu + [(I - bi')\hat{c}]p_1\} = \{t_0^3, t_0^{10}, t_0^{11}\}. \qquad (1)$$

It increases μ, π, b and c by $\Delta\mu$, $\Delta\pi$, Δb and Δc, respectively, and recalculates e from (1) above with these increased values to produce a new expenditure vector e_1.

It forms

$$\Delta e_1 = e_1 - e_0 = \{\Delta t^3, \Delta t^{10}, \Delta t^{11}\}.$$

The calculation of the elements of the expenditure vector b was carried out by a separate series of programs. Experiments were made on linear, quadratic and more complicated forms of the basic expenditure system. Each of these programs used a two-stage iterative least squares process, similar to that used in probit analysis.

Stage 2

This stage calculates the fourteen vectors concerned with final demand. These are:

the final demand for consumers' goods and services

$$t^{1\cdot 3} = A^{1\cdot 3}t^3$$

and the change in this final demand

$$\Delta t^{1\cdot 3} = A^{1\cdot 3}\Delta t^3,$$

the final demand for the replacement of consumer durables

$$t^{1\cdot 10} = A^{1\cdot 10}t^{10}, \text{ and } \Delta t^{1\cdot 10},$$

the final demand for extensions to consumer durables

$$t^{1\cdot 11} = A^{1\cdot 11}t^{11}, \text{ and } \Delta t^{1\cdot 11},$$

the final demand for industrial replacement

$$t^{1\cdot 8} = A^{1\cdot 8}t^8, \text{ and } \Delta t^{1\cdot 8},$$

the final demand for current government purposes,

$$t^{1\cdot 4} = A^{1\cdot 4}t^4, \text{ and } \Delta t^{1\cdot 4}$$

the final demand for government capital replacement,

$$t^{1\cdot 12} = A^{1\cdot 12}t^{12}, \text{ and } \Delta t^{1\cdot 12}$$

and the final demand for government capital extension

$$t^{1\cdot 13} = A^{1\cdot 13}t^{13}, \text{ and } \Delta t^{1\cdot 13}.$$

It forms and stores the four import vectors

$$\begin{aligned} t^{15\cdot 3} &= \hat{a}^{15\cdot 3}t^3, \\ \Delta t^{15\cdot 3} &= \hat{a}^{15\cdot 3}\Delta t^3, \\ t^{15\cdot 4} &= \hat{a}^{15\cdot 4}t^4, \\ \Delta t^{15\cdot 4} &= \hat{a}^{15\cdot 4}\Delta t^4, \end{aligned}$$

the final output vector

$$f^* = t^{1\cdot 3} + t^{1\cdot 4} + t^{1\cdot 8} + t^{1\cdot 10} + t^{1\cdot 11} + t^{1\cdot 12} + t^{1\cdot 13} + t^{1\cdot 15}$$

and $\Delta f^* = \Delta t^{1\cdot 3} + \Delta t^{1\cdot 4} + \Delta t^{1\cdot 8} + \Delta t^{1\cdot 10} + \Delta t^{1\cdot 11} + \Delta t^{1\cdot 12} + \Delta t^{1\cdot 13} + \Delta t^{1\cdot 15}$.

Finally it calculates the two import constants

$$i't^{15\cdot 3} + i't^{15\cdot 4}, \text{ and } i'\Delta t^{15\cdot 3} + i'\Delta t^{15\cdot 4}.$$

Subroutine 1

Given the vectors, z, a_2, a_3 and $a^{5\cdot 1}$, and the matrix $A^{1\cdot 1}$, this subroutine forms the matrix,

$$Z = A^{1\cdot 1} + (I + \hat{a}^{5\cdot 1})a_2 a_3'. \qquad (2)$$

It checks that the column sums of the matrix Z are all < 1.

Starting from $x_0 = 0$, it uses an iterative process to calculate

$$x_{t+1} = Zx_t + z, \qquad (3)$$

and it tests if

$$y = \sum_{i=1}^{n} |x_{t+1,\, i} - x_{t,\, i}| / \Sigma |x_{t,\, i}| < 10^{-7}.$$

It prints y at each cycle. This method, based on the expansion

$$(I - A)^{-1} = I + A + A^2 + \ldots,$$

was used in preference to the normal inversion subroutines, so that experiments could be made with non-linear forms of the model.

The convergence test in this subroutine must be tighter, that is to say the final constraint must be smaller, than that used in the main loops of the program, or the whole process breaks down.

Thus here $y < 10^{-7}$, when in the main loops $y < 10^{-6}$.

Stage 3

(3A) This stage forms the vector

$$\Delta f^{**} = \Delta f^* - (I + \hat{a}^{5\cdot 1})[\Delta a_1 + a_2 \{(i'\Delta t^{1\cdot 15} + i'\Delta t^{3\cdot 15}) - (i'\Delta t^{15\cdot 3} + i'\Delta t^{15\cdot 4}) - \Delta\beta\}].$$

This is a first approximation of the change in the final demand $\Delta\beta$. The program sets up the vectors z, a_2 and a_3, to enter subroutine 1, for the first time, so that $z = \Delta f^{**}$ and Z is given by (2) above. It solves (3) for the vector

$$x_0 = (\Delta T^{2\cdot 1})_{(0)}'i,$$

which gives a first approximation to the change in total output of commodities $(\Delta T^{2\cdot 1})'i$.

(3B) The program forms the vector

$$\Delta t^{15\cdot 2} = \hat{a}_3(\Delta T^{2\cdot 1})'i,$$

which is the change in industrial imports, the scalar

$$i'\Delta t^{15\cdot 1} = (i'\Delta t^{1\cdot 15} + i'\Delta t^{3\cdot 15}) - (i'\Delta t^{15\cdot 3} + i'\Delta t^{15\cdot 4}) - \Delta\beta - i'\Delta t^{15\cdot 2},$$

which is the total change in competitive imports, and the two vectors

$$\Delta t^{15\cdot 1} = \Delta a_1 + a_2 i'\Delta t^{15\cdot 1}$$

and $$\Delta t^{5\cdot 1} = a^{5\cdot 1}\Delta t^{15\cdot 1},$$

which are the detailed changes to be expected in competitive imports and in indirect taxation.

The program forms the vector of change in the final demand,

$$\Delta f = \Delta f^* - (I + \hat{a}^{5\cdot 1})\Delta t^{15\cdot 1},$$

and enters subroutine 1, for the second time, with

$$z = \Delta f,\ a_2 = a_3 = 0, \text{ so that } Z = A^{1\cdot 1}.$$

It calculates the next approximation $x_1 = (\Delta T^{2\cdot 1})_{(1)}'i$, and then it tests if

$$y = \sum_{i=1}^{31} |x_{1i} - x_{0i}| / \Sigma |x_{1i}| < 10^{-6}.$$

It prints y, and if $y > 10^{-6}$, it repeats the first loop from (3B), using $x_1, x_2, \ldots$ in place of $x_0, x_1, \ldots$. When $y < 10^{-6}$, the program goes on to the next part of the calculations, using the latest approximation, x_n.

(3C) This part forms the five vectors

$\Delta t^2 = C_0(\Delta T^{2\cdot 1})'i$	(change in output)
$t^9 = \hat{k}_0(\Delta t^2)$	(capital extensions in industry)
$t^{1\cdot 9} = A^{1\cdot 9}t^9$	(final demand for capital extension in industry)
$t^{1\cdot 7} = \hat{a}^{1\cdot 7}(\Delta T^{2\cdot 1})'i$	(final demand for capital commodities)
$t^{15\cdot 7} = \hat{a}^{15\cdot 7}t^{1\cdot 7}$,	(capital commodity imports).

It forms the first approximation f^{**} to the final demand b, where

$$f^{**} = f^* + t^{1\cdot 7} + t^{1\cdot 9} - (I + \hat{a}^{5\cdot 1})[a_1 + a_2 \{(i't^{1\cdot 15} + i't^{3\cdot 15}) - (i't^{15\cdot 3} + i't^{15\cdot 4}) - \beta - i't^{15\cdot 7}\}],$$

and enters subroutine 1, for the third time, with $z = f^{**}$, and a_2, a_3 set so that again Z is given by (2) above.

This time the subroutine calculates the first approximation to total commodity output

$$x_0 = (T^{2\cdot 1})_{(0)}'i.$$

(3D) This forms the vector of industrial imports

$$t^{15\cdot 2} = \hat{a}_3(T^{2\cdot 1})_{(0)}'i$$

and the scalar of total competitive imports

$$i't^{15\cdot 1} = (i't^{1\cdot 15} + i't^{3\cdot 15}) - (i't^{15\cdot 3} + i't^{15\cdot 4}) - \beta - i't^{15\cdot 2} - i't^{15\cdot 7}.$$

It forms the vectors

$t^{15\cdot 1} = a_1 + a_2(i't^{15\cdot 1})$,	(competitive imports)
$t^{5\cdot 1} = \hat{a}^{5\cdot 1}t^{15\cdot 1}$	(indirect taxes)

and $f = f^* + t^{1\cdot 7} + t^{1\cdot 9} - (I + \hat{a}^{5\cdot 1})t^{15\cdot 1}$, (final demand).

It enters subroutine 1, for the fourth time, with $z = f$, $a_2 = a_3 = 0$, so that $Z = A^{1\cdot 1}$, and it calculates the next approximation

$$x_1 = (T^{2\cdot 1})_{(1)}'i.$$

It tests if $y = \sum_{i=1}^{31} |x_{1i} - x_{0i}| \Big/ \sum_{i=1}^{31} |x_{1i}| < 10^{-6}$.

It prints y and if $y > 10^{-6}$, it repeats the second loop from (3D), using $x_1, x_2 \ldots$ in place of $x_0, x_1, \ldots$. It goes on to the next part of the calculation, when $y < 10^{-6}$, with the latest approximation x_n as $(T^{2\cdot 1})'i$.

It forms the vector of total industrial current output

$$t^2 = C_0(T^{2\cdot 1})'i.$$

It prints all the thirty-one vectors in the store, t^3 to $t^{15\cdot 4}$, listed above.

It forms and prints the two matrices

$$T^{2\cdot 1} = C_0\hat{x}_n \quad \text{where } x_n = (T_{2\cdot 1})'i,$$

and

$$T^{1\cdot 2} = A^{1\cdot 1}\,.\,(T^{2\cdot 1}),$$

and two vectors, which are the row and column sums of $T^{1\cdot 2}$.

Comparative speeds

On EDSAC 2, the three stages were programmed in machine code and run separately. Each stage took over twenty minutes, so that one complete calculation took about one hour of machine time. In 1962, the author reprogrammed the calculations for the Ferranti Orion, in Extended Mercury Autocode (E.M.A.). One complete calculation took about 25 minutes on that machine. The saving arose mainly from the fact that the calculation was no longer split into three parts. We transferred this program to the Manchester Atlas in February 1963, and at that time, we repeated the calculations for 1970 with $\alpha = 0\,(5\%)\,50\%$. Each run took about five minutes including compiling time. Last year, the calculation was reprogrammed for the Elliott 503 in Elliott ALGOL. Here one run took about seven minutes, excluding compiling time. Recently I have repeated the calculations on the Cambridge Titan with the new Titan Autocode compiler. Including compiling time, it now takes 110 seconds for one run.

Conclusions

Professor Stone is at present engaged on a comparison between various existing models of total economies. The first conclusion to emerge from this work is that our present model is much simpler in logical structure than most of its contemporaries. This is probably due to the fact that we wanted an actual numerical realization of our model and not simply a theoretical structure.

On the other hand, our model employs a larger and more detailed set of data. Also it uses rather sophisticated estimation processes, and these experiments could not be attempted at all without the use of a fairly fast computer.

The second conclusion is that our ability to perform the actual calculations has now far outstripped our abilities to collect and adjust the data and to refine our experimental model.

The prime function of this model is as a set of experimental tools, any one of which can be varied quickly and easily, so that we can produce results from several different structural hypotheses, using the same set of data.

Our work is not intended as a "crystal ball" to be used in prediction, although the trends calculated here are at least as likely to prove to be true as those produced by any other means.

Full details of the economic structure of the model and of the results of all these calculations have been published from time to time in *Programme for Growth*, Richard Stone and others, Chapman and Hall (Vols. I–VI) (1962–66).

Book Review

Computer and Information Sciences, by J. T. Tou and R. H. Wilcox, 1965; 544 pages. (London: *Macmillan*, 110s.)

This symposium contains a number of papers on pattern recognition, adaptive control, perception-type devices and automata theory. There is nothing about computer software, nor about computer hardware (apart from some speculations by Ledley about the use of very large parallel computers). Machine intelligence work in the Newell–Shaw–Simon, McCarthy–Minsky tradition is represented by a single, not very rewarding paper by Kuck and Krulee. Information theory occurs only incidentally and information retrieval not at all.

Apart from one or two trivia almost all the papers are heavily mathematical in tone. The little boxes and arrows so dear to the early cybernetricians have largely been replaced by integral signs, which is encouraging. But few of the papers report the results of experimental work or computer simulations. There is still a tendency to present plausible algorithms and justify them by hand simulation of a toy problem.

Of the twenty-three papers I can only remark on a few of the more interesting ones.

Ross Ashby's keynote address "The next ten years" is worth quoting: "Thus the difficulty with artificial intelligence is not to get it, but to get enough of it."

An impressive paper by Rosenblatt, "A model for experiential storage in neural networks," proposes a perception mechanism for recording a sequence of stimuli and later recalling them so as to reproduce the responses with a high probability. An expression for the probability of correct recall is derived mathematically and numerical evaluation of this suggests that complete recording of experience may be within the capability of the human brain. Rosenblatt suggests how several such devices could be coupled to produce something analogous to a computer running through a sequence of instructions.

Block, Nilsson and Duda in "Determination and detection of features in patterns" give a lucid exposition of the usefulness of describing patterns in terms of features and of two methods of determining good sets of features. It is a pity that they give no results of applying their algorithms to non-trivial pattern recognition tasks. Many people would have found this more interesting than their formal proof of convergence.

In a previous paper in the *IBM Journal* Kamentsky and Liu described an efficient character recognition program which determined a good set of recognition features. Here they give a theoretical treatment and compare expressions for the error rates with the experimental results.

Widrow and Smith review work at *Stanford University* on the "Adaline," an adaptive threshold element. Networks of these elements have been trained to perform tasks in fields as varied as speech recognition, weather forecasting and control of a dynamic system (the broom balancer).

For 110s. one might have expected better proof reading. Pages 42–45 are mis-numbered and exchanged with 46–49 and on page 85 the argument is obscured by total confusion between F and $\bar{F}$.

The general standard of the papers is quite high and the book is worth a place in any library concerned with this field.

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