

Control data for the variable step length integration method

```

STEP IN PRINTING = .1
Prints outputs at intervals
of .1 sec integration time.
MAXIMUM INTEGRATION TIME = 2
ERROR = .01 Maximum error is set to
1%.
HEADING
TIME OUTPUT
* *
(TIME) 1, 1
PRINT (F2) 1, 3
COMPLETE
    
```

9.2 Time delay example

The equation of non-linear growth is:

$$\frac{dX(t)}{dt} = AX(t) - BX(t)X(t - \tau)$$

The graphs of Fig. 5 are for solutions of the above equation having the following constants:

$$A = B = 1$$

$$\tau = 1.8.$$

9.3 Function generation by switches

A simple example is provided by generating a saturation curve represented by three straight line segments as in Fig. 6.

The function of Fig. 6 may be simulated using two switches operating at levels  $K_1$  and  $K_2$ . The block diagram for representing the curve is given in Fig. 7.

9.4 Simple control system optimization

The simple example dealt with in Section 9.1 may be adapted to give an example of optimization. The original unit F1 may be replaced by a similar unit

having a variable parameter  $W$  in place of the coefficient value 2. A value of the parameter  $W$  can then be found which gives a response having a minimum value of the error squared integral ( $\int (\text{error})^2 dt$ ).

The performance criterion is incorporated into the block diagram of Fig. 8 and the data specification for determining the optimum value of  $W$  is given below.

SIMPLE EXAMPLE OPTIMIZED

```

* *
I → A1.1
F2 → A1.2
A1 → F1
F1 → F2
A1 → M1.1
A1 → M1.2
M1 → F3

* *
A1
(1, -1)
F1
(10)/(P + W)
F2
(1)/(P)
M1
(1)
F3
(1)/(P)

* *
STEP LENGTH = .05
MAX INTEGRATION TIME = 4
INITIAL VALUE OF W = 8
OPTIMIZE (W) 1, 3
MINIMIZE (F3) 1, 3
COMPLETE
    
```

Book Review

*Introductory Numerical Analysis of Elliptic Boundary Value Problems*, by Donald Greenspan, 1965; 164 pages. (London: Harper and Row Ltd., 52s.)

This book gives an account of the use of finite difference methods for the numerical solution of boundary value problems. The author has given a careful treatment of the setting up of the finite-difference formulae including the approximation of normal derivatives at points on curved boundaries. Equations in both two and three independent variables are considered. However, there is no mention of the popular techniques which apply when the differential operator is written in divergence form.

Convergence theorems are proved for the usual second-order equations, and in one case an interesting example of non-uniqueness is given. The scope of these results is conventional enough. However, the author gives only extremely crude information about the rate of convergence of the

finite-difference approximations, and a whole range of very important related material (from the treatment of singularities to the deferred approach to the limit) is omitted.

Also unsatisfactory is the sketchy summary of numerical methods for solving the resulting sets of algebraic equations. The author is apparently convinced that the optimum value of the over-relaxation parameter is 1.8 in all circumstances. However, this aspect of the subject has already been well documented. The list of references has been prepared with a great deal of energy, but little discretion.

The most attractive feature of the book is the detailed treatment of topics often neglected in the past (a chapter on nonlinear equations is included in addition to the material already mentioned). For this reason, in particular, the reviewer feels the book can be studied with profit, although he cannot recommend it as a self-contained introduction to the subject.

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