Discussion and results

Several points should now be made in connection with the problem in question. First, the reduction of the determinantal equation to an explicit polynomial is not necessary and, indeed, leads to a more ill-conditioned problem by the reduction to a fewer number of coefficients (from the array to the polynomial), and introduces possible inaccuracy problems. Scaling problems, magnified by a data-condensation process, may be handled easily in the original array if, in fact, such difficulties exist initially. Although the computations for the results below were computed using complex arithmetic (in an existing IBM 7044 computer program), the same results could have been obtained for this case with real variables since no complex conjugate roots are present to establish symmetry problems in applying the real secant method. The values of the determinant were obtained using elimination methods, again due to the usage of a general-purpose program, whereas the inclusion of an initial reduction for Hyman's method is helpful in solving ordinary eigenvalue problems with this method (note, however, that this initial reduction also reduces the number of coefficients for the problem).

Using this method the following results are obtained and appended to Rachman's table for the parameters and matrix given in his paper.

The data in the last column of **Table 1** were computed in single-precision arithmetic (36 bits, 27 for mantissa) with no preconditioning nor scaling of the array. Total computation time was approximately 30 seconds on the IBM 7044. It should be noted that the trace (as Rachman points out, not a sufficient accuracy test) as calculated here is in excellent agreement with the true value; we also add that the nine-digit numbers given in the table were obtained in converting eight-digit reduced eigenvalues to the tabled values, and the ninth digit is retained only to aid in immediate comparison to previous data.

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To the Editor,
The Computer Journal.

Timetabling and scheduling problems

Sir,

Some of the difficulties associated with computer application to general class timetables and scheduling are well known, and more aspects have recently appeared in an airline study which support Elizabeth Barraclough's comments (this *Journal*, Vol. 8, p. 136) that the choice for a best method for compiling timetables seems to lie between a theoretical and a manual method. The need she mentions for some parameters of the input data related to the "degree of success" of the timetable is also supported. Apart from this the study has a cautionary bearing on the type of linear programming analysis used by Dr. R. E. Miller (*Domestic Airline Efficiency —An Application of Linear Programming*, M.I.T. Press 1963).

In a scheduling and timetabling study made for the recent introduction of Boeing 727 aircraft into Australian domestic airlines it was required to schedule initially two 727 aircraft (and later three), between five capital cities in Australia daily under the following conditions:

(i) Transcontinental flight Sydney–Perth via Adelaide preferably daily;
(ii) Daily return Sydney–Brisbane, Sydney–Adelaide;
(iii) No positioning (empty) flights allowed and one aircraft to end at Sydney and one at Melbourne each evening;
(iv) First flights to be between Melbourne and Sydney and not earlier than 7.45 a.m.;
(v) No flights later than 9.00 p.m. and allowance made for time differences coast to coast;
(vi) Allowance for ground time at airports;
(vii) Allowance for all routing patterns and possible connecting flights of the 727 aircraft to achieve the specified inter-capital service;
(viii) What effect of time for some alternate daylight servicing of aircraft;
(ix) Maximum flights between the major business centres Melbourne and Sydney;
(x) Annual flying hour usage for each aircraft to be a maximum up to 3,750 hours.

Manual trial and error methods were started by experienced airline operations staff while the computer method was developed. In the latter, a binary notation of routing was used using a series of binary δ's which finally allowed 16 routing possibilities to be designated by variables which had to be either zero or unity.

The single case of δ_1 will illustrate the method which led into integer programming formulation to cover routing possibilities in the timetabling problem:

$\delta_1 = 1$ designated a return flight Sydney–Brisbane prior to Sydney–Adelaide (direct or via Melbourne depending on δ_2)

(*continued on p. 77*)

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Correspondence (*continued from p.* 66)

$\delta_1 = 0$ designated a flight or flights Sydney–Melbourne and Melbourne–Sydney prior to Sydney–Adelaide flight (direct or via Melbourne depending on δ_2).

Other variables such as the number of flights between any points were required to be integral.

The resulting non-linear integer programming problem was eventually reduced to a series of linear integer problems and run on a Honeywell H-400 6-tape computer.

The initial results appeared excellent at first sight, with annual usages of 3,701·4 hours and 3,688·4 hours. However, the time of loading of passengers at Adelaide for Sydney was not satisfactory to the airline both from the point of view of poor loading and "customer goodwill." Savings could be made in the degree of daylight servicing provided in the schedule, but the other factors were more important and this reflects back to Elizabeth Barraclough's desire for parameters in the input to relate to the "degree of success" of a timetable.

At this stage, it appeared that to rectify matters and incorporate other commercial judgements, it would be difficult to quantify them all within the memory confines of even very large computers. Matters such as mathematically specifying the nature and interdependence of the frequency of a service and the expected passenger traffic arose. The program was modified to avoid some unacceptable time periods. Memory size quickly limited any further conditions being incorporated.

The results showed that the departure time Sydney–Perth was critical but it was observed that, with several cases, it would be possible to improve this transcontinental service to one flight every day by extracting one daily Sydney–Melbourne—Sydney flight at a low loading time, and using the overall weekly time gained to give almost exactly the time required to achieve seven flights per week to Perth instead of five. This was without disturbing the remainder of the schedule, and was a matter which was apparent by some human observation but not apparent to the computer method. By the time the computer study was completed, the results agreed with the best manual schedule derived independently, and it was verified by the machine that the optimum had been reached for this case.

An additional point which arose seems worthy of mention based on the experience gained regarding formulating commercial judgements: the desirability of parameters in the input related to the degree of success and the advantage of some human intervention. It may be that Dr. Miller's type of linear programming study which indicated very excessive operating costs including "overscheduling", may have suffered due to problem simplification rather than have overscheduling costs introduced in explanations. The only factors which would seem to contribute to operating costs—low load factors, low utilization and uneconomic aircraft—would cover "overscheduling" and were discussed in the study.

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