$c[n-1] := b \times d[n-2];$
$d[n-1] := t; d[n] := c[n];$
$c[n] := a \times t;$
$c[n+1] := (m+n) \times (2 \times n - 1) \times d[n]/((m-n) \times (2 \times n + 1));$
**comment** *this is the n.f. for the nth polynomial*;
**end**

**Correspondence** (*continued from p.* 320)

This is always possible because $U$ is similar to a symmetric matrix. The solution of (2) when $l = 0$ is

$$w(t) = \left[\exp\left(-\frac{1}{(\Delta x)^2} Ut\right)\right] c$$

$$= \sum_{i=0}^{N} \alpha_i z_i \exp\left(\frac{-\lambda_i t}{(\Delta x)^2}\right) \qquad (7)$$

or
$$w(n\Delta t) = \sum_{i=0}^{N} \alpha_i z_i [\exp(-r\lambda_i)]^n. \qquad (8)$$

On the other hand the solution of (4) with $k = 0$ is

$$v^n = \{[I + r\theta U]^{-1}[I - r(1-\theta)U]\}^n c \qquad (9)$$

$$= \sum_{i=0}^{N} \alpha_i z_i \left(\frac{1 - r(1-\theta)\lambda_i}{1 + r\theta\lambda_i}\right)^n. \qquad (10)$$

The replacement of (2) by (4) therefore replaces each factor $\exp(-r\lambda_i)$ in one time step of (8) by a factor

$$\frac{1 - r(1-\theta)\lambda_i}{1 + r\theta\lambda_i} \qquad (11)$$

whence the result follows.

This simple relationship between the stability properties of equations (2) and (4) does *not* necessarily persist when Crank and Nicolson's procedure with $\theta = \frac{1}{2}$ is applied to a non-linear or non-autonomous problem (Rosenbrock and Storey, 1965, pp. 173–175). Some formulae giving improved stability and truncation error have been suggested in an earlier note (Rosenbrock, 1963).

**References**
PARKER, I. B., and CRANK, J. (1964). "Persistent discretization errors in partial differential equations of parabolic type", *The Computer Journal*, Vol. 7, pp. 163–167.
KEAST, P., and MITCHELL, A. R. (1966). "On the instability of the Crank–Nicholson formula under derivative boundary conditions", *The Computer Journal*, Vol. 8, pp. 110–114.
CRANK, J., and NICOLSON, P. (1947). "A practical method for numerical evaluation of solutions of partial differential equations of the heat-conduction type", *Proc. Camb. Phil. Soc.*, Vol. 43, pp. 50–67.

ROSENBROCK, H. H., and STOREY, C. (1965). *Computationa Techniques for Chemical Engineers*, pp. 8–15 (Pergamon Press).
ROSENBROCK, H. H. (1963). "Some general implicit processes for the numerical solution of differential equations" *The Computer Journal*, Vol. 5, pp. 329–330.

Yours faithfully,

H. H. ROSENBROCK

Control Systems Centre,
University of Manchester,
Institute of Science and Technology,
Sackville Street, Manchester 1.
16 June 1966.

*To the Editor,*
*The Computer Journal.*
Sir,
I should like to reply to the letter by K. Wright (this *Journal*, May 1966, p. 115) about my paper entitled "Error curves for Lanczos' 'selected points' method" (this *Journal*, January 1966, p. 372). I apologise for stating that Wright's statement about the form for the residual (this *Journal*, January 1964, p. 358) is incorrect. His letter clearly shows the source of my confusion.

However, I do not agree with the simpler derivation in the letter for the form of the residual. Although

$$r(x) = \dot{e}(x) - e(x)\frac{\partial F}{\partial y} \cdots$$

there is no justification in dropping the whole right-hand side except for the first term. The derivation given in my paper based on the Picard iteration does show how errors build up.

Incidentally, there are two typographical errors in my paper. In Table 4, $H_{31}$ should read $-0.089142227$ instead of $-0.08142227$. In Table 5, $G_{41}$ should read $0.37699459$ instead of $0.376994519$.

Yours sincerely,

W. KIZNER

Jet Propulsion Laboratory,
California Institute of Technology
4800 Oak Grove Drive,
Pasadena,
California 91103
1 August 1966