By evaluating the most complicated integral C_4, for a sequence of values of H starting with $-0\cdot95$, and using both the recurrence relations (53), . . ., (56) and Gaussian integration formulae of various orders, it was found that, using floating-point arithmetic with a precision of about 9 decimal places, the integrals could be obtained correct to at least 6 significant decimal places by using the analytical formulae for $H < -0\cdot4$ and $H > 0\cdot5$, and for $-0\cdot4 \leqslant H \leqslant 0\cdot5$ using the Gauss formula of order 5.

6. Discussion

The methods described in this paper have been incorporated in a computer program which has been used by a number of civil engineers. A description of the practical aspects of the program and of some of the problems associated with it will be reported elsewhere. The method of specifying the data for an analysis is similar to that for the slip-circle program described by Little and Price (1958).

It is well known that the Newton–Raphson technique may be used for solving non-linear simultaneous equations, and it is known that extra controls on the variables are often essential to make the process converge. The control (*c*) described in § 3, that the sum of the squares of the residuals should be reduced by each iteration is of general application for one or more non-linear equations. It is less arbitrary than the control (*b*) of § 3 which is advocated in N.P.L. (1961) which requires an arbitrary fixed limit to the change, or proportional change, of any of the variables. However, it should be noted that squares of the residuals should be scaled if necessary so that they are of the same order of magnitude, otherwise the restriction can increase the number of iterations considerably. The scaling may conveniently be performed by dividing each residual by the sum of the squares of its derivatives with respect to the unknowns.

7. Acknowledgement

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Correspondence

To the Editor,
The Computer Journal.

Sir,

Many algorithms have been published in which values of π, 2π, π^2 etc., are written as constants, commonly with about 9 significant figures. Sometimes comments are included that these constants are intended as approximations to π etc., and that the number of significant figures should be adjusted to the capacity of the computer.

But it is not necessary to use any explicit approximation to π or related values, since in ALGOL 60 the appropriate value can be generated by the statement

$$\text{pi} := 4 \times \arctan(1)$$

and the variable pi may then be used as a component of arithmetic expressions. The accuracy with which pi approximates to π is limited only by the accuracy of the arctan function (and possible roundoff). For instance, in FORTRAN II-D on the IBM 1620, the corresponding statement PI = 4·0 * ATANF (1·0) (with 28 decimal digits in the mantissae of floating-point numbers) does indeed give π correctly to 28 significant figures. Likewise, of course, if e is required then the statement $e := \exp(1)$ can be used.

Yours faithfully,
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